European Summer University on History and Epistemology in Mathematics Education

esu 10

20-24 july 2026
Aveiro, Portugal

Book of abstracts

universidade de aveiro
dmat department of mathematics
ECIU university

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About

ESU10

The **European Summer University on the History and Epistemology in Mathematics Education** (ESU) is originally the initiative of the French Mathematics Education community of the IREMs, in the early 1980's. From those meetings emerged the organization of a Summer University on a European scale, in close cooperation with the HPM group.

Financially supported by National Funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., under CIDMA-UA with grants UID/04106/2025 (https://doi.org/10.54499/UID/04106/2025) and UID/PRR/04106/2025 (https://doi.org/10.54499/UID/PRR/04106/2025) and under the CIDTFF R&D Unit, with the reference UID/00194/2025 (https://doi.org/10.54499/UID/00194/2025).

This event will be certified as a training course for Portuguese teachers (groups 230 and 500). Finally, this event has the institutional support from UNESCO Portugal.

The program and activities of ESU-10 are structured around the following more general topics:

1. Integrating history and historical epistemology of mathematics in mathematics education.
2. Integration of the history of mathematics in classrooms (curricula, courses, textbooks, experiences, original historical sources and material of all kinds).
3. History of mathematics in (pre-service and in-service) teacher education.
4. Mathematics and its relation to science, technology, and the arts: Historical issues and socio-cultural aspects in relation to interdisciplinary teaching and learning.
5. Topics in the history of mathematics education.
6. Mathematics and cultures.
7. History of mathematics in Portuguese-speaking countries (Portugal, Brazil, Mozambique, Cape Verde, Angola, Sao Tome e Principe, Guinea-Bissau)

The official languages of ESU-10 are **English**, **Portuguese** and **French**.

- All plenary lectures and panel discussions will be in English.
- It is preferable to organize workshops in English. Nevertheless, workshop organizers who intend to organize their workshop in another language are advised and encouraged to prepare copies in English of the material to be distributed to the participants (e.g. slides, worksheets etc). This will certainly increase participation, as well as, facilitate communication among participants.
- Oral presentations can be delivered in any of the official languages. However, for presentations not in English, presenters will be asked to use two sets of slides; one set in the language they are going to give their presentation, and one set in English.

Timetable

Overall Time Schedule

(provisional – 18/03/26)

	Monday 20	Tuesday 21	Wednesday 22	Thursday 23	Friday 24
9.00 – 10.00		**PL** **J. M. Delire**	**PL** **K. Karpińska**	**PL** **A. Bernard**	**PL** **F. Romero-Valhonesta**
10.00 – 10.30	Registration	Break (2)	Break (4)	Break (5)	Break (7)
10.30 – 11.00		**WS 2 (2)**	**Panel (M. N. Fried, org.): The Place of History and Pedagogy of Mathematics in Mathematics Education Research**	**WS 2 (3)**	**OP7** **OP8** **OP9** **OP10**
11.00 – 11.30	Opening				
11.30 – 12.00	**PL** **E. Barbin**				
12.00 – 12.30					
12.30 – 14.00	Lunch	Lunch	Lunch	Lunch	Lunch
14.00 – 14.30	**WS 2 (1)**	**OP1** **SOP1** **OP2** **SOP2**	**PL** **H. Leitão**	**WS 1,5** (2)	**OP11** **SOP3** **OP12** **SOP4**
14.30 – 15.00					
15.00 – 15.30		**OP3** **OP4**	Social Event		**PL** **S. Lawrence**
15.30 – 16.00				Break (6)	
16.00 – 16.30	Break (1)	Break (3)		**OP5** **OP6**	Closing
16.30 – 17.00	**PL** **S. Gessner**	**WS 1,5** (1)			
17.00 – 17.30					
17.30 – 18.00	**Round Table** Recreational Mathematics: from history to current education + Mathematical Circus			**WS 1,5** (3)	
18.00 – 18.30		Walking Sunset			
18.30 – 19.00				Walking Sunset	
20.00-…			**Conference Dinner**		

Monday, 20 of July

Auditório Carlos Borrego
10:00-11:00 Registration
11:00-11:30 Opening Session

11:30-12:30 Évelyne Barbin
European Summer Universities (1993-2025): more than thirty years of sharing

Restaurant
12:30-14:00 Lunch

Departamento de Matemática (Dpt. of Mathematics)

14:00-16:00 Workshop		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Poleni's geometrical machines to construct transcendental curves: a chance to explore the materiality of mathematical concepts today* (Pietro Milici, Frédérique Plantevin)
11.1.31	1	*A Mathematical Toolkit for Middle School, Inspired by Archaeology and History* (Farid Kherbouche, Azmiya Padavia)
11.1.30	1	*Teaching the impossible: squaring the circle in 19th-century geometry textbooks* (Carène Guillet)
11.1.29	2	*Ferrari, Tartaglia, and the Fixed-Opening Compass, or How to Reconstruct the Elements by Changing the Third Postulate* (Riccardo Bellé, Veronica Gavagna)
11.1.28	2	*Women's Voices in the History of Mathematics Education: Learning with Agnesi, Everest Boole, and Chisholm Young* (Paola Magrone, Antonio Cigliola, Maria Giulia Lugaresi, Elena Scalambro)
11.1.27	4	*Portugal as a playground for geometric fortification during the Restoration War (1640–1668)* (Frédéric Métin)

16:00-16:30 Break

Auditório Carlos Borrego
16:30-17:30 Samuel Gessner 4
The Local and Global History of Early Modern Mathematics: Material Culture as a Key

17:30-19:00 Round table + *Mathematical Circus* – Jorge Nuno Silva (Org.)
Recreational Mathematics: from history to current education

Tuesday, 21 of July

Auditório Carlos Borrego

9:00-10:00 Jean Michel Delire 6

Indian mathematics, a source for a globalized history of mathematics

Departamento de Matemática (Dpt. of Mathematics)

10:00-10:30 Break

10:30-12:30 Workshop		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Experimenting the tangibility of Tangent with Historical Mechanisms* (Pietro Milici, Frédérique Plantevin)
11.1.31	1	*Examining the history of mathematics as a Gateway Towards Inclusivity, Engagement, and Understanding in the Mathematics Classroom: Understanding the Irish secondary school context. A report on research in progress* (Stephen Begley, Diarmaid Hyland, Ciarán Mac an Bhaird)
11.1.30	1	*The Making of the Greatest Maths Book in the World: A workshop on bringing episodes in the history of mathematics to life in the classroom by means of theatre, incorporating a sequence of short playlets set in ancient Greece* (Gavin Hitchcock)
11.1.29	2	*Mathematics Across Cultures: Historical Examples as Dialogical Spaces for Understanding, Meaning, and Agency* (Shafie Shokrani, Fatemeh Ahmadpour)
11.1.28	3	*Logarithms: A History of Simplification* (Teresa Clain, Helena Santos)
11.1.27	1	*Solving problems from the* Arithmetica Practica, y Speculativa *by Juan Pérez de Moya (1562), in secondary education* (Iolanda Guevara Casanova, Fàtima Romero Vallhonesta)

Restaurant

12:30-14:00 Lunch

Departamento de Matemática (Dpt. of Mathematics)

13:50-15:00 Short oral presentation		
Room	**Top.**	**Title and Authors**
11.1.26	1	*Lazare Carnot: Mathematics Teacher's Specialized Knowledge attached to History and Epistemology* (Francisco Djnnathan da Silva Gonçalves, Etienne Lautenschlager)
	1	*Past and present in mathematics didactics as a tool in teacher training* (Stela Segev, Tsurit Elitzur)
	1	*The Influence of Historical Mathematics Content on Elementary Students' Learning Interest: A Focus on Integers* (Tzu-Hsuan Tseng, Wei-Chin Wu)

	3	*Historical sources as mediators for conscious teaching Styles - A meta didactic analysis of Fibonacci's "Broken Numbers"* (Silvia Cerasaro)
	3	*Use of historical primary sources to support preservice teachers' insight into mathematical ideas* (Erica Minuz, Marianne Thomsen)
	4	*Walking with Archimedes* (Laura Isolani, Barbara Cennini, Mariacarmela Ando, Attiio Ferrini)
	4	*Dante, Mathematics and Poetry in the* Divine Comedy *: Games & Maths – An interdisciplinarity approach* (Irene Bonciani, Gianmario Marrelli, Gabriele Pasquini, Attilio Ferrini)

Departamento de Matemática (Dpt. of Mathematics)

14:00-16:00 Oral Presentation		
Room	**Top.**	**Title and Authors**
11.1.32	1	*All around the Parallel Postulate* (Anna Petiurenko, Piotr Błaszczyk)
	5	*Les brevets d'invention de matériel pédagogique : un regard nouveau sur la réforme des mathématiques modernes (1950-1980)* (Thomas Préveraud)
	2	*The Representation of the History of Mathematics in Turkish High School Textbooks: A Comparison of the 2019-2024 Curriculum and the 2025 Türkiye Century Maarif (Education) Model* (Öznur Kılıçkaya)
	1	*Didactic Resource for the Evolution of Fundamental Concepts: From Euclid to the Crisis of Foundations* (Francesca Coppa, Silvia Cerasaro, Anna Amirante)
11.1.31	2	*A contribution to the teaching of negatives from texts by G. Cardano and M. Stifel* (Anne Boyé)
	2	*The Use of the History of Mathematics as a Pedagogical Approach* (Anabela Monteiro, Maria Paula Oliveira, Dina dos Santos Tavares)
	2	*Bridging the Conceptual Gap: Integrating History of Mathematics and GeoGebra to Enhance Understanding of the Fundamental Theorem of Calculus* (Tung-Shyan Chen)
	2	*Developing Mathematical Skills: The Challenge of History of Mathematics in the Classroom* (Paulo Gil)
11.1.30	2	*Iconographic approach for studying Apollonius'* Conics (Silvia Lanaro)
	2	*Mathematical machines as historical artefacts: epistemological meanings and mathematical competencies in a laboratory approach to the parabola* (Federica Troilo, Elisabetta Giuseppina Erione, Roberto Capone)
	2	*Problèmes inspirés par l'histoire des mathématiques : la valeur formatrice de l'analyse des sources historiques d'hier pour comprendre comment pensent les élèves d'aujourd'hui* (Fernando J. Bifano, Nicolás E. Igolnikov)
	2	*Recreational mathematics in technical education during modern mathematics reform* (Alexandra Sofia Rodrigues, Corália Pimenta)

11.1.29	3	*Conceiving a history of mathematics course for pre-service mathematics teacher education in Hungary* (Katalin Gosztonyi)
	3	*Exploring the impact of historical mathematical contexts on critical thinking in secondary education* (Larisa Sali)
	3	*Histoire des mathématiques et patrimoine culturel local : une expérience de formation des enseignants à Mayotte* (Jean-Berky Nguala, Dominique Tournès)
	3	*History of mathematics in teacher education Amsterdam University of Applied Sciences* (Peter Lanser, Desiree Agterberg)
11.1.28	5	*Educational Change Mediated through Textbooks: A Comparative Analysis of 3+1 Mathematics Textbooks* (Dionisis Baltzis)
	6	*The Culture of Sangaku in Japan During the Edo Period* (Noriko Tanaka)
	6	*Mathematics in Cultural Context: AI Reflections on Ethnomathematics* (Gizem Ünsal, Seyma Sengil-Akar, Elif Saygı)
11.1.27	4	*Algebra in Image: Dialogues Between Art and Science during Bolognese Renaissance* (Ana Paula Pereira do Nascimento Silva, Valdenize Lopes do Nascimento, Amanda Pereira do Nascimento Silva)
	4	*College Students' Responses to the Implicit Mathematical Narrative in Two Historic Paintings* (Po Hung Liu)
	4	*Planck's 1900 paper as a resource for an interdisciplinary laboratory with secondary school students* (Maria Giuseppina Adesso, Roberto Capone, Oriana Fiore)
	4	*The history of symmetry – from art, via crystallography, to mathematics* (Franka Miriam Brückler)

16:00-16:30 Break

16:30-18:00 Workshop		
Room	**Top.**	**Title and Authors**
11.1.32	2	*Exploring Historical-Mathematical Tasks: From Textbooks to Classroom Practic* (Marc Moyon, Katia Badet-Borri, Sanae Echatoui, Pascal Rouffignac)
11.1.31	1	*Analog Tools Before the Digital Age: Rediscovering Historical Geometry Instruments in the Classroom* (Jakub Michal)
11.1.30	2	*A C-OW and four Pouillys: a variety of uses of local resources from elementary school to teacher training* (Frederic Metin, Clara Philippe)
11.1.29	3	*Inexhaustible heritage of Johannes Kepler: three inspirations for high school mathematics* (Filip Beran)
11.1.28	2	*The Multipurpose Tangent Solver: A Hands-On Journey through the History of Calculus* (Anita Lugli, Michela Maschietto, Pietro Milici)
11.1.27	3	*Proof with History* (Nélia Amado, Helmuth Malonek)

11.1.26	2	*Problematizing the Pythagorean Theorem through the Practices of the Pythagorean Community* (Bruna Moustapha-Corrêa, Aline Caetano da Silva Bernardes & Marcello Amadeo)

Wednesday, 22 of July

Auditório Carlos Borrego

9:00-10:00 Karolina Karpińska 5

Mathematics Education in Secondary Schools for Boys in 19th-Century Poland: Schools with Polish, Prussian, Austrian, and Russian Curricula

10:00-10:30 Break

10:30-12:30 Panel *The Place of History and Pedagogy of Mathematics in Mathematics Education Research*, Michael N. Fried (Org.), with Jaime Carvalho e Silva, Renaud Chorlay, and Tinne Hoff Kjeldsen, Helena Durnová.

Restaurant

12:30-14:00 Lunch

Auditório Carlos Borrego

14:00-15:00 Henrique Leitão 7

Mathematics in Early Modern Portugal: The challenge of the sea

15:00-18:00 Social Event TBA

20:00-... Conference Dinner TBA

Thursday, 23 of July

Auditório Carlos Borrego

9:00-10:00 Alain Bernard 3

History of mathematics for future teachers, in a nutshell

Departamento de Matemática (Dpt. of Mathematics)

10:00-10:30 Break

10:30-12:30 Workshop		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Reviving Forgotten Mathematics: The Case of Gaspar Nicolas's* Tratado da Pratica d'Arismetica (Teresa Costa Clain, Ana Patrícia Martins, Marc Moyon, Hélder Pinto)
11.1.31	1	*Mental Representations of Ascending Order around the Plagiograph* (Rainer Kaenders, Ysette Weiss)
11.1.30	5	*The Royal Gymnasium of Lissa and its Jewish mathematics teacher Julius Tœplitz* (Henning Heller)
11.1.29	2	Homem que Calculava*: A Paradoxical Historical Source with Multiple Translations and a Didactic Tool* (Pierre Ageron, Marine Lebreton, François Plantade)
11.1.28	3	*Sobre a sequência e as extensões de Leonardo e os números figurados gregos: propriedades e sua interpretação para sala de aula por meio de Tabuleiro e ladrilhos* (Francisco Regis Vieira Alves, Milena Carolina dos Santos Mangueira, Francisco Evamar Barros)

Restaurant

12:30-14:00 Lunch

Departamento de Matemática (Dpt. of Mathematics)

14:00-15:30 Workshop		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Understanding ratio through cultural connections: braiding Euclid's* Elements, *Book V hóroi to a narrative account of primordial measure* (Isabella Fascitiello, Ana Millán Gasca, Francesca Neri Macchiaverna)
11.1.31	1	*Criteria for the didactical transposition of historical mathematical knowledge: Their analysis, relevance, and applicability in the case of solving cubic equations and introducing the imaginary numbers* (Constantinos Tzanakis, Yannis Thomaidis)
11.1.30	2	*Arithmetic with Papy's minicomputer* (Wendy Gœmans, Dirk De Bock)
11.1.29	2	*Bringing the History of Mathematics into the Classroom* (Ho Ming Jason Yip, Thomas K. Briggs)

11.1.28	2	*Luisa Volterra D'Ancona's Biomathematical Research as a Resource for Mathematics Education* (Erika Luciano, Marina Marchisio Conte, Sara Omegna, Elena Scalambro)
11.1.27	3	*Getting into mathematical, historical and pedagogical questions through videoclips* (echoing Bernard's plenary lecture) (Alain Bernard, Azmiya Padavia)
11.1.26	3	*Un exemple de mise en œuvre de la méthodologie du groupe AHMES: former à l'enseignement du périmètre et de l'aire du disque* (Frédéric Laurent, Jean-Marc Pilandon)

15:30-16:00 Break

16:00-17:00 Oral Presentation		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Potential of Collaboration between History and Mathematics Teachers: Inter/Transdisciplinary approach* (Abdelrahman Affan, Michael N. Fried)
	1	*Propositions to Sharpen Young Minds from York to Singapore* (Pedro Freitas, Andreia Hall, Tiago Hirth, Sónia Pais, Jorge Nuno Silva, Ricardo Teixeira)
11.1.31	1	*Exploring the Integration of Artificial Intelligence in a History of Mathematics for Educators Graduate Course* (Richard Velasco)
	1	*Fagnano's problem and the epistemological role of geometry in eighteenth century optimization problems* (Elisabetta Erione, Roberto Capone, Maria Giuseppina Adesso, Oriana Fiore)
11.1.30	2	*The Articulation between History of Mathematics, Digital Technologies, and Argumentation: A Teaching Experiment Proposal Using a Diagram from Savasorda's Book of Geometry* (Leverson Batista Gabriel de Campos, Adriana de Bortoli)
	2	*The Educational Value of the History of Mathematics for Learners with Intellectual Disabilities* (Elena Gil Clemente, Raquel García Catalán, Elisa Passacantilli)
11.1.29	3	*How Quido Vetter (1881-1960) made history of mathematics an inherent part of mathematics teaching and teacher training in Czechoslovakia?* (Helena Durnova)
	3	*La place de l'histoire des mathématiques dans la formation des futurs enseignants du secondaire au Maroc* (Nisrine Lahlil)
11.1.28	4	*The role of AGDs in proving old problems* (Nélia Amado)
	5	*Spatial Thinking and Geometry in the Colombian Mathematics Curriculum: A History of the Present* (Luis Carlos Vargas Zambrano)
11.1.27	7	*Tensions dans l'élaboration des documents curriculaires dans les années 1990: les relations entre le Brésil et la France* (Sidnéia Almeida Silva)
	7	*Vector Calculus in Brazil and France (1900-1930): A historical-mathematical perspective based on pioneering works* (Sabrina Helena Bonfim)

17:00-18:30 Workshop		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Some reflections on the role of the table as an artifact in the mathematics classroom based on historical examples* (Greisy Winicki Landman)
11.1.31	2	*An escape room on ancient Indian combinatorics* (Desiree Agterberg)
11.1.30	2	*Binary System and Boolean Algebra: A Teaching Journey through History and Manipulation* (Daniela Tossini, Elena Scalambro, Giovanni Longo)
11.1.29	2	*Historical sources and classroom culture: Obstacles and opportunities concerning* Liber abbaci *in secondary school* (Laura Tomassi)
11.1.28	2	*Inverse Tangent Constructions in Dynamic Geometry: A Formula Free Approach* (Pietro Milici, Paola Magrone, Corrado Falcolini)
11.1.27	3	*Comparing versions of sources as part of a master course in history of mathematics for teacher students* (Bjørn Smestad)
11.1.26	3	*Histoire des mathématiques à l'université Marie et Louis Pasteur (Franche-Comté, France): un exemple de pratique* (Hombeline Languereau)

Friday, 24 of July

Auditório Carlos Borrego

9:00-10:00 Fàtima Romero Vallhonesta 2

The importance and challenge of incorporating original mathematical texts in the classroom

Departamento de Matemática (Dpt. of Mathematics)

10:00-10:30 Break

10:30-12:30 Oral presentation		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Doing analytical geometry is not like playing a tune by turning a barrel-organ* (Évelyne Barbin, René Guitart)
	1	*From Pascal to Leibniz: Intuition in (re)discovering Calculus* (Remus Titiriga)
	1	*History of Mathematics and Mathematics Education: Key Themes, Dialogues, and Provocations* (Fernando Bandeira Figueiredo)
	1	*History of mathematics in the classroom: the risk of an incomplete pedagogy* (Adriano Dematté)
11.1.31	2	*For a classroom in-depth study of the knowledge of the number e and the exponential function* (Emilia Florio, Giuseppe Canepa, Elda Guala, Leonardo Primavera, Giuseppina Fenaroli)
	2	*From MathPods to PodsMath: The History of Mathematics as a Catalyst for Inclusive and Accessible Digital Learning* (Rafaela Silveira Salvajoli Leite, Adriana de Bortoli, Rafael Hamamura)
	2	*From Research to Practice and Back: A Teacher Education Course on Integrating History of Mathematics* (Aline Caetano da Silva Bernardes, Bruna Moustapha-Corrêa)
	2	*The effect of solving historical Fibonacci problems on students' beliefs about the nature of mathematics* (Claudia Éthel Figueroa Suárez, Josip Slisko Ignjatov)
11.1.30	2	*Historical sequential movements (HSM) as a way of approaching mathematics in school* (Iran Abreu Mendes)
	2	*Using Historical Sources to promote reflections on Rigor and mathematical Reasoning in High School: Area Determination as Case* (Nynne Milthers Gjerlufsen, Britta Eyrich Jessen, Tinne Hoff Kjeldsen, Amanda Koertz Wedderkopp)
	2	*"6 Milestones", an Erasmus+ project to promote historical recreational problems in mathematics education* (Lisa Rougetet, Tiago Hirth, Fotis Lazarinis, Monika Musilek, Giota Mourettou, Maurizio Giaffredo, Jorge Nuno Silva)
11.1.29	3	*Reviewing Descartes' construction of curves by continuous movement with secondary teachers, using a DGS* (Veronica Hoyos)
	3	*The Borel exhibition at the IHP: historical and educational issues* (Alain Bernard, Laurent Mazliak)

	3	*The History of Mathematics as a Critical Tool in Pre-service Teacher Education in Brazil* (Carla Regina Mariano da Silva)
11.1.28	5	*Teachers' personal collections and the Math Education: the case of Rio de Janeiro in the second half of the 20th century* (Carlos Mometti)
	5	*Teaching mathematics in the 17th century. Pierre Hérigone's Mathematical Course* (Pedro José Herrero Piñeyro, Antonio Linero Bas, Antonio Mellado Romero)
	5	*Teaching mathematics with machines: reconstructing Poleni's pedagogical activity via his lecture notes* (Davide Crippa)
	5	*The Rule of Three in Recknekonsten – Hans Larsson Rizanesander's treatment of problems of proportion* (Johanna Pejlare, Reza Hatami)
11.1.27	4	*Shaping Czech Engineering Mathematics in the early 20th century: The Curík–Lerch Conflict* (Jan Kotulek)
	4	*Art, Vision, and Knowledge: The Legacy of Alberti's Perspective in Mathematics Education* (Débora Wagner, Cláudia Flores)
	4	*"Mathematics for the Million" according to Bento de Jesus Caraça* (Jaime Carvalho e Silva)
11.1.26	6	*Seventeenth-Century Contributions to the Historical Development of the Method of Separation of Variables in Differential Equations* (Anna Karla Silva do Nascimento)
	6	*Co-production of knowledge and training of indigenous teachers in the Amazon* (Cristiane do Socorro dos Santos Nery)
	6	*Effect of hand games in enhancing the algebraic thinking skills of Native American high school students* (Sijo Varghese)
	6	*Mathematics in Cultural Context: AI Reflections on Ethnomathematics* (Gizem Ünsal, Seyma Sengil-Akar, Elif Saygı)

Restaurant

12:30-14:00 Lunch

Departamento de Matemática (Dpt. of Mathematics)

14:00-15:00 Oral presentation		
Room	**Top.**	**Title and Authors**
11.1.32	1	*Meeting halfway: How can History of mathematics be a resource for mathematics education research?* (Renaud Chorlay)
	1	*One way in which scholarly historical knowledge in complex analysis is nuanced in contemporary Mexican university contexts* (José Gerardo Piña-Aguirre)
11.1.31	2	*History of mathematics in pre-service secondary education teacher training in Brazil and Spain. A comparative analysis and implications* (Antonio M. Oller-Marcén, Thiago Pedro Pinto)

	2	*How the History of Mathematics Can Support the Discovery of an Original Proof: The Case of the Law of Sines* (Abdellah El Idrissi)
11.1.30	2	*Who's Who in Mathematics? Integrating the History of Mathematics into the Classroom through Gamification* (Marisabel Antunes, Cristina MR Caridade, Verónica Pereiras, Ana Paula Aires, Jaime Carvalho e Silva)
	3	*The use of rhetoric and syncopated algebra in pre-service teacher education* (Ladislav Kvasz)
11.1.29	7	*The history of Mathematics Education in Teacher-training of Secondary Education in Cabo Verde* (Mario Dos Santos Fernandes)
	7	*Genèse du Calcul Vectoriel à l'École des Mines d'Ouro Preto (Brésil) au début du XXe siècle* (Davidson Paulo Azevedo Oliveira)
11.1.28	5	*The "GIRP", An international sect of New Math enthusiasts in the post-New Math era?* (Dirk De Bock,Wendy Goemans)
	5	*Ubiratan d'Ambrosio at the Brazilian colloquium of mathematics (1957): interlocutions, knowledge, and the constitution of mathematics education in Brazil* (Juliana Chiarini)
14:00-15:00 Short oral presentation		
11.1.27	2	*Analysis and Synthesis in teaching Calculus: Historical insights* (Patrìcia Nunes da Silva, Silvana Batista da Silva)
	2	*Euclidean principles of area equivalence for geometric problems to work with undergraduate and high school students* (Fernando Antonio de Araujo Carneiro)
	2	*Repunit sequence in teaching* (Líviam Santana Fontes)
	5	*Imaginary Quantities and Philosophical Legitimacy: Apolinar Fola's Contribution to 19th-Century Spanish Science* (Juan Pons Gonzàlez, Ana María Lluch Persi, Maria Santagueda Villanueva)
	5	*Letters, Networks, and Institutionalization: Ubiratan D'Ambrosio's Correspondence and the Creation of SBHMats* (Relicler Pardim Gouveia)
	5	*Transnational Interactions of Ubiratan D'Ambrosio During the Formulation of the Course Trends in Mathematics Education: The Circulation of Knowledge and the Initial Proposal of This Discipline* (Alexandre Lauriano Copelli)

Auditório Carlos Borrego

15:00-16:00 Snezana Lawrence 1
History of mathematics for the Million

16:00-16:30 Closing Session

List of Abstracts

Plenary Lectures

Plenary lectures in the program are arranged following the numerical order of the theme (from 1 to 7, with a special lecture).

History of mathematics for the Million

Snezana Lawrence
United Kingdom

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In the Western world today, mathematics is often perceived as an abstract and daunting discipline, remote from everyday life and accessible only to a select few. This perception alienates many people and contributes to widespread mathematical anxiety. In this talk, I will address these challenges by advocating for the role of the history of mathematics in reshaping public attitudes.

Through exploring mathematical history, we can move beyond abstract concepts and algorithms to understand what it truly means to engage with mathematics. History allows us to see mathematics not merely as a body of knowledge, but as a human endeavour that has evolved alongside diverse cultures and historical periods. We can ask: What problems were mathematicians of the past trying to solve, and how did they approach them? What motivated their mathematical investigations? By examining stories from ancient Babylon to modern-day discoveries, we reveal mathematics as a living, dynamic process rooted in human curiosity and creativity.

But how does mathematics education feature these problems in general education? I will argue that to change this perception it is important to engage whole population and not only those who learn mathematics in schools and colleges, but their families also.

The principles I will discuss are drawn from my book, *A Little History of Mathematics* (Yale University Press, 2025), which spans 35,000 years of mathematical development, from prehistoric counting to cutting-edge research in the 21st century. Written with this public in mind, the book presents short, engaging episodes that showcase the diversity and richness of mathematical thought throughout history. My aim was to create a resource that could spark conversations between generations–encouraging teenagers, parents, and grandparents alike to reflect on how mathematics has shaped different civilizations and how its universal appeal allowed us to have a universal discipline underlying our communications and technologies from philosophy to software engineering and many other practical applications.

By grounding mathematical concepts in stories of real people and their challenges, we can inspire a more inclusive and appreciative view of mathematics. I will suggest further readings that build on these ideas, including Glen Van Brummelen's *The Mathematics of the Heavens and the Earth*, Jacqueline Stedall's *Mathematics Emerging*, and George Gheverghese Joseph's *The Crest of the Peacock*. Through such explorations, we can help demystify mathematics and reveal its profound human significance.

Snezana Lawrence is a historian of mathematics based in the UK, known for making mathematics engaging and accessible through her writings and educational work. Her books are *A Little History of Mathematics* (2025), *Mathematical Meditations* (2025), *A New Year's Present From a Mathematician* (2019), and *Mathematicians and Their Gods* (2015), each weaving historical insights with captivating storytelling. Beyond her publications, Snezana has significantly impacted mathematics education through her work on the history of mathematics.

She was on the advisory panel redesigning the National Curriculum in Mathematics in UK in 2014, arguing for inclusion of the history of mathematics; served as Chair of the History and Pedagogy of Mathematics International Study Group (2020–24) and was the first Education Officer of the *British Society for the History of Mathematics*. She is the Assistant Editor of the *British Journal for the History of Mathematics*, overseeing the educational submissions and contributes to international journals, including the *Nexus Network Journal*.

Since 2015, she has held visiting positions at Masaryk University (Czech Republic), the University of Lorraine (France), and the University of Kragujevac (Serbia). She also advises the University of Aveiro on research related to the history of mathematics.

The importance and challenge of incorporating original mathematical texts in the classroom

Fàtima Romero-Valhonesta 2

Universitat Politècnica de Catalunya (UPC), member of the History Group of the Associació de Barcelona per l'Estudi I Aprenentatge de les Matemàtiques, Spain

In teaching mathematics, we do not always convey to students the process of mathematical creation, how mathematical ideas evolve. We believe that mathematical instruction should include introducing students to the genesis of at least some mathematical ideas, lest they think that mathematical ideas or theorems arise spontaneously from privileged minds. One way of doing this is by way of original mathematical texts.

The difficulty of using original texts in the classroom is that not all historical texts or all sources are appropriate to design activities to implement in the classroom. What criteria should be used to select texts? Obviously, there should be some relationship to topics of the curriculum. Surely, our preferences will also play an important role, but no matter how interesting a text may be to us, we must think about how relevant it will be for students to learn the mathematical concepts involved. And how should we use these texts? Is it enough to read and comment on them in class to learn mathematics? For some texts, that may be the case, but for most of them it is necessary to add some guidelines for students to understand and in order to focus on the content we want them to learn. We can choose problems from ancient texts, present them to students and compare the ways they have found solutions with the way the author has resolved them. Or we can give students some excerpts that allow them to follow certain reasoning. Of particular interest are works written in the form of a dialogue, such as Plato's Dialogues and various works from the Renaissance.

In this lecture we will reflect on the importance of the history of mathematics for learning mathematics, specifically, on the criteria for choosing appropriate texts, that is, what characteristics a historical text must have for it to be helpful in learning mathematics. We will discuss how to implement such texts in the classroom, and we will present some examples from different topics and various disciplines such as probability or algebra.

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Fàtima Romero Vallhonesta has a degree in Mathematics from the Universitat de Barcelona (UB) and a PhD in History of Science from the Universitat Autònoma de Barcelona (UAB). She worked as a secondary school teacher at the Alexandre Satorras Secondary School in Mataró and then as an Inspector of Education for the Generalitat de Catalunya. These roles were compatible with giving Geometry classes to undergraduate students of Primary Education at the UAB. In addition, she gave Calculus classes at the Escola Tècnica Superior d'Enginyeria Industrial de Barcelona (ETSEIB). Now retired from her main job, she is a member of a Research Group of the Universitat Politècnica de Catalunya (UPC), and also a member of the History Group of the Associació de Barcelona per l'Estudi I Aprenentatge de les Matemàtiques.

Further, she is a member of the Museu de Matemàtiques de Catalunya (MMACA) with which she collaborates in some activities. Her research focuses on the history of mathematics through two main lines of investigation. The first one deals with the algebraization of Mathematics that took place from the 16th to the 18th century, specifically in the Spanish works written at the beginning of this period. The second area of investigation is related to the use of History of Mathematics in the school curriculum in order to find texts which can be instrumental in the learning of mathematical concepts.

History of mathematics for future teachers, in a nutshell

Alain Bernard

University of Paris Est Créteil, Centre Alexandre Koyré, France

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In one of his illuminating papers on the "use" of history of mathematics in the classroom Man Keung Siu (2007) posed a problem that was meant not only to reveal the concrete problems experienced by teachers when trying to introduce history into their courses and teaching but also to address the challenge of defining of what it means for him, as a teacher trainer, to dispel the underlying misunderstanding about what is at stake in this introduction of a historical perspective in one's teaching. How can one explain, in simple terms or with simple means, that the whole point is not about *knowing* history of mathematics, but about *understanding* its value and depth for key issues about the teaching of mathematics? In other words, Siu transformed a question from teachers, into a question for teacher trainers and historians.

These traditional questions are still fully relevant. They are furthermore reinforced today by the explosion of "teaching assistants" ranging from lists of references, video clips, AI apps, and online historical documents, all designed to assist effective teaching. In many cases these "aids" ultimately come to constitute a heavy set of constraints, essentially because providing "online resources" does not amount to creating the conditions to develop true and "living" resources (Trouche et alii, 2020). Although this question has long been explored by researchers in math education (Pepin et alii, 2017) and is well known to teacher trainers, in a world in which the lack of resources is apparently no longer a problem, we still need to ask how we can help people make the best of the host of "resources" available to them, especially when those resources are presented in a form that does not always facilitate their appropriation. Online resources for integrating history of mathematics in math teaching does not escape these dilemmas.

Being confronted, like anyone else, with these old and newer questions, I recently developed a videographic project with several colleagues and institutions (Bernard et alii, to be published). The purpose was to confront the above-mentioned dilemmas and offer a rich answer to the questions posed by some of my students about the quality of some video clips on history of mathematics available during the COVID crisis: what was their historical value and pedagogical interest? What could they do with this? The decision to produce our own series of videos clips initially followed a double intention. The first was to convey, in a *nutshell* – that is, within the hard constraint of 6 or 7-minutes clips – some kind of deep meaning about history of mathematics and its interest for teaching. The second was to provoke some kind of critical thinking about the videographic support itself. Beyond these initial intentions, it soon appeared that the collective creation of such video clips had an interest by itself, either to reflect or to trigger new questions. This project will thus serve me to illustrate the underlying issues related to the development of such "teaching aids", taking the form of a "nutshell", and meant to convey a sense of history about their chosen subjects.

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DAD-MULTILINGUAL.

Alain Bernard is assistant professor at University Paris Est Créteil in the institute for teacher training (INSPE). As historian of mathematics and mathematical sciences he works today on 18th cent. mathematics after a period in which he contributed to the history of late antique mathematics. He belongs to Centre A. Koyré (Aubervilliers) and contributes to several collective research teams, mainly the ENCCRE project (online critical edition of Diderot's and D'Alembert's *Encyclopédie*) as also the inter IREM group. For example, he published a "dossier critique" associated to the article "PROPOSITION, en mathématiques", in Édition Numérique Collaborative et CRitique de l'Encyclopédie, 2022 (available online at `https://enccre.academie-sciences.fr/encyclopedie/`).

The Local and Global History of Early Modern Mathematics: Material Culture as a Key

Samuel Gessner
Portugal

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Can a local museum inspire and challenge students of mathematics? The recreational side of mathematics has a long history – one that can be uncovered through museum heritage. This talk explores examples from the material culture of early modern mathematics, including manuscripts, early prints, and instruments. While mathematical concepts, techniques, and results can have global reach, mathematics has always been practiced within specific communities, places, and historical contexts. Local histories of mathematics offer a richer perspective on both the subject and the past of our own localities.

This talk presents a model of interdisciplinary collaboration that can be replicated anywhere. Since 2021, we have been developing a pilot project in Lisbon involving a mathematics teacher, museum staff, a product designer, and a historian of mathematics. Together, we have created low-cost models of historical instruments, inspired by authentic objects in local museums. These models serve both to engage students in mathematical reasoning and to foster inquiry into the mathematical past of their city.

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Samuel Gessner is an assistant researcher at the Center for History of Science and Technology (CIUHCT) and an invited professor at the Department of History and Philosophy of Science of the Universidade de Lisboa.
His research focuses on the diverse mathematical cultures in medieval and early modern Europe. He examines how they interacted by studying the role of mathematical and astronomical instruments as conceived by both theoreticians and practitioners. He emphasises using artefacts of material culture as primary sources, in particular mathematical and astronomical instruments, alongside textual documents.

Mathematics Education in Secondary Schools for Boys in 19th-Century Poland: Schools with Polish, Prussian, Austrian, and Russian Curricula

Karolina Karpińska
5
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At the end of the 18th century, Poland was partitioned three times. As result of the third partition in 1795, Poland vanished from the map of Europe, with its lands seized by three powers: Prussia, Russia, and Austria. Consequently, in the 19th century, schools in Polish territories operated under Polish, Prussian, Russian, and Austrian educational systems. This paper discusses the distinctive features of mathematics education according to which of these systems was implemented. Special attention is given to schools preparing students for matriculation examinations, such as gymnasia and real-type schools. Curricula, textbook contents, and sets of matriculation exam problems are characterized. Scientific publications by teachers are particularly valuable in this context, as they frequently discussed selected topics covered in schools. These publications provide insight into how teachers independently modified curricula to best prepare students for efficient functioning in everyday life as well as for university studies. Special attention is paid to the application of arithmetic and algebra in daily life, specifically, the so-called "citizen calculations", which include the calculation of pensions, annuities, and financial transactions related to banking. Moreover, in the 19th century, geometry was highly valued in Polish territories; depending on the period and whether Polish, Prussian, Russian, or Austrian curricula were in place, the focus varied between construction problems, surveying, analytical geometry, and descriptive geometry. It was believed that, alongside its practical applications, one of the main advantages of geometry was its ability to develop logical thinking skills, especially in the context of conducting complex, multi-step geometric constructions.

In the 19th century, secondary schools for boys preparing students for matriculation exams offered comprehensive curricula. Innovations in science were often quickly integrated into these programs. For example, in 1812, elements of descriptive geometry were introduced into schools with Polish curricula based on the works of Gaspard Monge and Jean N. Hachette. This advancement was made possible by a well-educated teaching staff – many secondary school teachers held doctoral degrees. Therefore, to provide a complete picture of mathematics education, this paper also highlights selected teachers, their professional qualifications, and their scientific and educational activities.

During the presentation, examples of 19th-century mathematics problems will be solved. The topics discussed in this paper may serve as material for contemporary teachers to incorporate elements of the history of mathematics education into their lessons.

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Karolina Karpińska is assistant professor in the Institute for the History of Science, Polish Academy of Sciences. In 2017, she received her PhD in the history of mathematics. Her current research is related to the history of mathematics education, with particular attention paid to the Polish territories in the 1795–1918. At that time, Poland was occupied by three empires: Prussia, Austria and Russia, and consequently there were schools with Polish, Prussian, Austrian and Russian curricula and relevant languages of instruction.

She has published several papers in this field, including: "Gnomonics in Secondary School Education in the Territories of Poland in the 17th–20th Centuries", in *Advances in the History of Mathematics Education* (Springer, 2022); " 'Denominate numbers' in mathematics school textbooks by Stefan Banach", *Historia Mathematica* 59(2022), and "Mathematics teaching at girls' Victoriaschule in Gdańsk from the mid-19th century until World War I", *Journal of Mathematical Behavior* (2025).

Indian mathematics, a source for a globalized history of mathematics

Jean Michel Delire
Belgium

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The mathematics developed in India is mostly unknown on this side of the Eurasiatic continent. Yet, the Indians elaborated, for more than three thousands years, several tools, sometimes comparable to those of contemporaneous civilizations. This is the case with the so called Pythagoras theorem, by instance, whose statement appears in the frame of the Vedic sacrificial ritual, elaborated more than one thousand years before Christ. The specific requirements of this ritual demonstrate geometrical knowledge which was collected in treatises, the Śulbasūtras, composed around Pythagoras'time. Other tools appeared in India long before they appeared in other civilizations, for example, the positional decimal system and negative numbers. In the first instance, the tendency to give to the powers of ten very different names, i.e. not built on the same root (as thousand), and the versification constraint in the composition of scientific texts explain the emergence of the positional system. In the second instance, the resolution of problems by numerical algorithms rather than by geometrical methods, allowed to give to negative numbers a status. These algebraic methods appeared in the first centuries of the Christian era, certainly before Āryabhaṭa, the first Indian mathematician-astronomer (5th c.) known by name.

In this lecture, I will develop these different points with the help of extracts from texts and iconography showing the role of Sanskrit, the language of the Indian literari. Indeed, this Indo-European idiom, akin to Greek and Latin, enabled Indian mathematicians to write equations before the word algebra appears in the famous book of al-Khwārizmī. The conditions for developing equations certainly already existed at Āryabhaṭa's time, as is shown by Bhāskara in his commentary to the Āryabhaṭīya (629). But the syncopated notation of these equations had especially been advanced by Brahmagupta in his *BrahmasphutaSiddhānta* (628) and by Bhāskarācārya in his entire work (12th c.). We shall note, in particular, the negative value of a number signaled by a dot above it, the arrangement in columns (second degree, first degree, fixed value) of the coefficients of an equation and the superposition of its two members, to represent an equality.

Of course, I will exemplify these points by pedagogical activities applicable in the classroom. Some of them have been tested in Belgian secondary schools, either on the occasion of the festival Europalia-India in 2013-2014 (see ESU 7 Acts, Copenhague), or by some of my students at the *Haute École de Bruxelles-Brabant* within the framework of their educational internship or their final dissertation.

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Jean Michel Delire holds a degree in mathematical sciences and a PhD in philosophy and literature from the University of Brussels (ULB), with a thesis on the oldest Sanskrit texts with mathematical content. This thesis was published by Droz, Geneva, in 2016.
For fifteen years now, he has focused his research on the mathematical and astronomical works of Raja Savai Jai Singh II (1689-1743) of Jaipur, Rajasthan, where J.M. Delire frequently resides, as well as on the contributions of Jesuit missionaries to the understanding of Indian science during the same period.
J.M. Delire has taught mathematics at the high school level and the history of mathematics at the university level. He is currently lecturing a course on History of Mathematics and on Science and Civilization of India – Sanskrit Texts, at the Institute of Advanced Studies of Belgium at ULB. He edited *Astronomy and Mathematics in Ancient India* (2012) and *Art et Savoir de l'Inde* (2015), for the occasion of Europalia-India. He is also the author of *Mathématiques multiculturelles* (vol. I, 2018) and numerous articles.

Mathematics in Early Modern Portugal: The challenge of the sea

Henrique Leitão

7

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Mathematical activities in Portugal during the 15th to 17th centuries, both theoretical and practical, were profoundly influenced by the country's maritime expansion: transformations in mathematics occurred across social, institutional, intellectual, and symbolic domains. While the effects of oceanic navigation were initially evident in Portugal, similar developments occurred in all nations engaged in large-scale maritime enterprises, particularly Spain, England, and the Netherlands.

Technological demands of oceanic navigation soon highlighted the need for close collaboration between university-trained mathematicians and practical professionals, such as pilots, mariners, instrument makers, and cartographers. A wide array of new questions in astronomy, cosmography, cartography, and instrument-making required the expertise of skilled mathematicians. As a result, mathematical talent was redirected toward solving the novel challenges posed by seafarers and cosmographers. The social landscape for the practice of mathematics was significantly altered due to these demands: circles of mathematical experts working with mariners emerged; new institutions were established to facilitate these collaborations; novel programs for the mathematical education of maritime personnel were implemented; and mathematical consultants were employed by the crown, the aristocracy, and commercial enterprises.

The influence of maritime exploration was also felt at the theoretical level. Long-distance voyages brought about considerable transformations and advancements in mathematics. A new, mathematically-based "science of navigation" had to be developed, a deeper understanding of the geometry of nautical charts was required, and substantial changes were made in mathematical cartography. The exploration of the Earth on a planetary scale gave rise to critical new problems in cosmography and cartography. Groundbreaking concepts, such as the loxodromic curve and Mercator's projection, were directly linked to this new maritime reality.

Perhaps the most profound impact was felt at even deeper levels. Mathematics gained a newfound prominence within the hierarchy of knowledge, and the awareness of its practical utility was significantly heightened.

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Henrique Leitão is Senior Researcher at the Department for the History and Philosophy of Science at the University of Lisbon, Portugal. His research interests are centered on the history of 16th-17th cent. cosmography and cartography. He is the author or editor of more than 20 books; the most recent (with J. M. Moreno Madrid), is: Drawing the Gateway to the Pacific. Maps, Charts and Other Visual Representations of the Strait of Magellan, 1520-1671. He is member of various learned societies, including the Lisbon Academy of Sciences and has been awarded several academic prizes and distinctions. In 2018 he received an Advanced Grant by the European Research Council.

European Summer Universities (1993-2025): more than thirty years of sharing

Évelyne Barbin Special PL
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1993 was the year of the first ESU in Montpellier (France). There were 244 participants in Montpellier, coming from France and 29 other countries. It was organized by the French inter-IREM Committee "Epistemology and History of Mathematics," and it was the fifth French Summer University after Le Mans (1984), Toulouse (1986), La Rochelle (1988), Lille (1990). ESU meetings are marked in three distinct ways: they are open to secondary teachers, they welcome interdisciplinarity, and they have three official languages: English, French, and the language of the ESU host country. After 1993, ESU meetings were organized in Portugal, Belgium, Sweden, Czech Republic, Austria, Denmark, Norway and Italy, thanks to colleagues who accepted to be organizers. In my talk, I will come back to all the meetings and to their success along the 30 years of ESU.

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She co-organized 20 colloquia, 8 summer universities and the first European Summer University (ESU) "Epistemology and History in Mathematics Education" in 1993. She was co-chair of nine ESU from 1996 to 2024. Since 1980, she is a member of the "International Study Group on the Relations between History and Pedagogy of Mathematics" (HPM). She was chair of HPM from 2008 to 2012 and HPM-2012 in D

Oral Presentation

Oral presentations in the program are arranged alphabetically by title.

A contribution to the teaching of negatives from texts by G. Cardano and M. Stifel

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"Cardan was also the first to notice the multiplicity of values of the unknown in equations, and their distinction into positive and negative. This discovery which, with another by Viète, forms the basis of all those made by Harriot and Descartes on the analysis of equations, this discovery, I say, is clearly contained in his *Ars Magna*. [...] These negative roots, he calls them feints." (Montucla 1758, *Histoire des mathématiques*, vol. 1, p. 871) Girolamo Cardano's best-known work is the one commonly referred to as *Ars Magna*, published in 1545, where precisely these "negative" solutions appear, not mentioned and even rejected in one of his previous writings, the *Practica arithmetice* of 1539, which was intended to be the sum of mathematical knowledge of that time, following the example of his predecessor Luca Pacioli's *Summa de Arithmetica* (1494). By trying to reconnect Pacioli to Cardan, via Michael Stifel's *arithmetica Integra* (1544), we will follow the beginnings of this familiarisation with negatives that will remain negative quantities for a long time. This study will make it possible to highlight the importance of the notion of binomials such as $3 - \sqrt{2}$, for example, in the need to define the rules for operations on negatives, in particular the rule of signs for multiplication. Rule that will perhaps logically lead Cardan to give two solutions for such equation as $x^2 = 4$, namely 2 and -2, and then, not without difficulty, to justify the acceptance of several solutions for an equation, including negatives ones. It will take several centuries, we know, for these negatives to become numbers, but the study of these texts can shed a new light on their teaching which is still problematic. The students of the 21st century are not lacking in concrete images for these numbers, but the operations on the negatives, the relationship of order, are always significant obstacles, even for students from the most advanced classes. We will present some elements illustrating our reflection on this teaching.

Presented texts

- Cardan Jérôme (1539) *Practica arithmetice et mesurandi singularis*, Milan, Castellioneus J. A
- Cardan, Jérôme (1576) *De vita propria*, trad. Dayre, J., 1991, Paris, Belin.
- Cardan Jérôme (1663) *Artis magnae sive de regulis algebraicis*, *Hieronymi Cardani opera omnia*, volume 4, Lyon, J. Aat Huguetan et M. Ant.Ravaud.
- Pacioli Luca (1464) *Summa de Arithmetica*, *geometria*, *de proportioni et de proportionalita*, Venise, Paganino de Paganini.
- Stifel, Michael (1544) *Arithmetica integra*, pref. Melanchthon Philippus, Nuremberg, J. Petreium.

The Use of the History of Mathematics as a Pedagogical Approach

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2
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Mathematics is often perceived as a demanding and demotivating subject by both students and their parents. This perception, rooted in pedagogical practices that privilege the mechanical application of formulas and procedures, distances learners from the true nature of Mathematics. It becomes evident that the absence of context and meaning in classroom tasks limits student engagement and compromises the quality of learning. Several authors have highlighted the potential of the History of Mathematics as a didactical resource to foster conceptual understanding, motivation, and a more humanised view of the discipline (Fauvel, 1993; Roque, 2012; Aires & Costa, 2021).

This work proposes an alternative approach centred on the use of the History of Mathematics as a pedagogical tool. It presents a set of classroom tasks inspired by mathematical challenges that date back many centuries, inviting students to explore the origins of the theories they study today, the motivations behind their development, and the ways in which each concept emerged as a response to a concrete problem.

By contextualising mathematical content and giving voice to the historical narratives that underpin it, this approach aims to promote more meaningful learning, spark students' curiosity, and foster a more positive relationship with the subject. The study also presents the results of implementing these tasks in real classroom settings, analysing students' responses, the impact on their learning and motivation, and the level of interest generated among different learner profiles. The analysis of written work, classroom interactions, and qualitative feedback reveals a clear increase in engagement, greater curiosity toward non-routine problems, and improved conceptual understanding–particularly when students recognise the connection between a concept and the historical problem that originated it. A reduction in mathematics anxiety was also observed, along with a greater willingness to discuss strategies, compare historical and contemporary methods, and justify reasoning. The findings suggest that integrating the History of Mathematics into classroom practice is an effective pedagogical strategy for promoting deeper learning, humanising the discipline, and strengthening students' connection to mathematical knowledge. This approach emerges as a promising pathway for enriching teaching practices and fostering a more positive, meaningful, and culturally informed relationship with Mathematics.

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Roque, T. (2012). História da Matemática, uma visão crítica desfazendo mitos e lendas. Zahar.

Algebra in Image: Dialogues Between Art and Science during Bolognese Renaissance

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Based on the painting *L'Algebra* (1572) by Giovanni Battista Ramenghi, also known as Bagnacavallo the Younger, this paper presents reflections, guided by Luis Radford's Theory of Objectification, on the relationships between mathematics, science, and art within the context of the Bolognese Renaissance. Starting from the premise formulated by the Theory of Objectification (Radford, 2016, 2021) that a work of art constitutes an artifact that incorporates, refracts, and reaffirms the ideologies of its time, we propose an analysis of Ramenghi's painting informed by this theoretical perspective. As argued by Lopes (2022), such an approach is grounded in the understanding that there exists a dialectical relationship between the artwork and the context in which it was produced, characterized by a mutual and continuous interaction: the context provides the artwork with the ideological content that permeates it, while the artwork, in turn, reaffirms and reconfigures the ideologies that enabled its creation. Ramenghi's painting illustrates the two-way relationship between science and art: on the one hand, the painting contributes to disseminating and consecrating the scientific novelty represented by Bombelli's treatise (1572); on the other, science offered new themes and intellectual challenges to artistic practice. In this regard, to what extent can Ramenghi's *L'Algebra* be understood as an instrument of public diffusion and legitimation of the algebraic knowledge systematized by Bombelli – It is important to emphasize that the painting in question was conceived based on a previously established iconographic model–most likely *Arithmetic* (1595) by Cornelis Cort, one of a series of seven engravings representing the liberal arts. Iconographic analysis reveals the explicit presence of volumes identified as parts of Bombelli's work, as well as representations of figures and mathematical formulas derived from his treatise. These elements suggest that Ramenghi, beyond adopting conventional models of the liberal arts, conferred upon algebra its own dignity. In this sense, the painting functions as a vehicle for the diffusion and legitimation of a new scientific knowledge, contributing to the consolidation of algebra's autonomy.

Ramenghi's painting holds historical value, and its interpretation, supported by other evidence, allows for consideration of a shift in the perception of algebra during that period. The analysis of the painting reveals that the artist not only reinterpreted a traditional iconographic model but also transformed visual language into an instrument of scientific affirmation. This convergence between art and science makes it possible to understand Bombelli's work as a milestone in the construction of algebra's epistemological identity: not merely as a technical discipline, but as a cultural expression of a new way of thinking about mathematics.

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All around the Parallel Postulate

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1

Textbooks in geometry are often insufficiently precise about the axiomatic foundations of Euclidean geometry with respect to two theses: Euclid's Fifth Postulate and Proposition I.32, which states that the sum of the angles of a triangle equals two right angles. These theses are, however, equivalent under the Archimedean axioms (Hartshorne 2000, pp. 321-322; Bonola 1912, pp. 118-121). The workshop aims to provide a set of exercises that develop intuition for the distinction between the Parallel Postulate and the thesis of Euclid's Proposition I.32.

We therefore discuss the Parallel Postulate in Euclid's formulation (*Elements*, Postulate 5), Euclid's proof of Proposition I.32, and Hilbert's version of the parallel axiom (Playfair's axiom).

We then point out further applications of the Parallel Postulate throughout the Elements, specifically in Propositions I.46 (the construction of a square), I.47 (the Pythagorean theorem), III.21 (the angle in a semicircle is a right angle), and IV.5 (the existence of a circle circumscribing a triangle). This part of the workshop can readily engage participants, since the propositions discussed are well known; however, we will emphasize the question of whether, instead of the Parallel Postulate, one can rely solely on Proposition I.32.

We introduce Saccheri's theory by examining the equivalence between the existence of Euclidean triangles (that is, triangles whose angles sum to two right angles) and the existence of squares. In fact, parts of this argument are applied in school geometry in other contexts. We sketch Saccheri's proof that the existence of a Euclidean triangle implies the Parallel Axiom, and we emphasize the role of the Archimedean axiom in that proof. Returning to the *Elements*, we show that Propositions I.46, I.47, and III.21 are equivalent to the existence of a Euclidean triangle, in the sense that they do not exploit the full strength of the Parallel Postulate. We also show that Proposition IV.5 (the existence of a circle circumscribing a triangle) occupies a different position. On this basis, we proceed to introduce a specific non-Euclidean geometry known as semi-Euclidean geometry. In this geometry, all triangles have angle sums equal to two right angles; however, the Parallel Postulate fails.

A model of semi-Euclidean geometry is based on Cartesian geometry over the hyperreal numbers. Unlike models of hyperbolic geometry, in this model straight lines are straight lines in the usual sense, and angles are angles between straight lines (Błaszczyk, Petiurenko 2021). The prerequisites for this part of the workshop are rather modest: familiarity with Cartesian geometry over an ordered field and a brief introduction to hyperreal numbers via the ultrapower construction.

For further information, other models of semi-Euclidean geometry are discussed in Hartshorne 2000, p. 305, and Greenberg 2007, pp. 213-214. Given this model, we elucidate the difference between, on the one hand, the Parallel Postulate and, on the other hand, Propositions I.32, I.46, I.47, and III.21. We also show that, in the presented model, Proposition IV.5 fails. We expect that this part of the workshop will be particularly beneficial for participants.

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Art, Vision, and Knowledge: The Legacy of Alberti's Perspective in Mathematics Education

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The act of looking transcends the simple apprehension of images, implying the ordering of the visible experience. According to Berger (1999), our mode of seeing is constructed by visual practices that affect and influence how we relate to and conceive of knowledge, including mathematical knowledge. Following this line of thought, this study examines the technique of central perspective as a historical and epistemological juncture that instituted a new rationality linking art, geometry, and the education of sight. We take Leon Battista Alberti's treatise, *De Pictura* (1435), as our starting point, where Alberti established the first rational and coherent application of an empirical theory of human vision to the art and technique of painting (Wagner, 2012). The analysis initially focuses on how Book One, fundamentally mathematical, articulates concepts and rules that support the elaboration of spatial representation as a window viewed by the spectator, fixed at a measurable point in space. The research investigates how the construction of the central perspective technique not only transformed artistic representation but also instituted a rational and geometric mode of seeing the world (Flores, 2007). The theoretical and methodological basis of this study is grounded in the genealogy of Foucault (2016), a strategy of analysis of relations, power dynamics, and practices that enables tracing and interrogating how this geometric logic consolidated itself as a regime of visibility in other cultural and technical practices, such as photography, architecture, and technical drawing. In each of these domains, the technique of central projection mediates the relationship between sight, representation, and knowledge, enabling the creation of more realistic and proportional spaces. Genealogy thus operates as the critical lens that problematizes central projection as a technology of savoir-pouvoir that translated spatial experience into a measurable and rationalized form. Based on this historical perspective, the discussion turns to the implications for mathematics education. It opens a space for the integration of artistic practices into mathematical learning, fostering creativity, spatial thinking, sensitivity to proportion and symmetry. Furthermore, it suggests that the history of perspective offers a lens for reflecting on how we come to "see mathematically," that is, how visualization intertwines with the constitution of mathematical understanding. In conclusion, by revisiting Alberti's central perspective technique and tracing its historical transformations, this work invests in the relationship between art and mathematics, believing that art can serve as a field for exercising mathematical thinking and vision. With this, we defend that art and history are fundamental fields of knowledge for the humanization of mathematical understanding, affirming the aesthetic and political dimension of knowledge.

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Bridging the Conceptual Gap: Integrating History of Mathematics and GeoGebra to Enhance Understanding of the Fundamental Theorem of Calculus

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Student underperformance in calculus at universities of technology remains a persistent pedagogical challenge, often attributable to insufficient mathematical preparation at the secondary level. Simultaneously, conventional calculus instruction–largely centered on procedural computation and examination-oriented practices–frequently fails to engage high-achieving students, resulting in disengagement and a limited perception of the subject's conceptual and practical

relevance. These challenges highlight the need for instructional approaches that address heterogeneous learning needs while fostering deeper mathematical understanding. This study proposes a pedagogical framework that integrates historical perspectives of mathematics with cooperative learning and technology-enhanced inquiry. Leveraging students' familiarity with mobile devices, the framework employs GeoGebra, a dynamic geometry environment (DGE), as a "modern compass and straightedge" to support visualization, exploration, and conceptual reasoning. The Fundamental Theorem of Calculus (FTC) is selected as the focal concept, given its central role in unifying differential and integral calculus. Although students often demonstrate procedural proficiency in differentiation and definite integration, they frequently lack an integrated understanding of the reciprocal relationship between instantaneous change and accumulated quantity. To address this gap, the instructional design reconstructs key historical insights–from Archimedes' method of exhaustion, through Barrow's geometric interpretation of tangency and quadrature, to Newton's concept of fluxions–as a coherent narrative scaffold for learning. These ideas are transformed into GeoGebra-based inquiry laboratories that promote active engagement and conceptual sense-making. The proposed approach aims to support students with weak foundations while extending learning opportunities for advanced learners, shift instructional emphasis from procedural fluency to conceptual reasoning, and strengthen the connection between calculus concepts and real-world applications. Overall, the model seeks to foster an inquiry-oriented learning environment that enhances students' attitudes toward calculus and improves learning outcomes.

College Students' Responses to the Implicit Mathematical Narrative in Two Historic Paintings

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In a general education course titled "Mathematics, Science, and Art in the Eastern and Western Renaissance," two historic paintings are introduced to explore the implicit mathematical narratives embedded within them. One is *Along the River During the Qingming Festival*, a handscroll by Zhang Zeduan from the Song dynasty in ancient China; the other is *The School of Athens*, a fresco by the Italian Renaissance artist Raffaello Sanzio da Urbino (Raphael). The theme of *Along the River During the Qingming Festival* centers on the everyday life of common people, yet several mathematical scenes are subtly depicted, including tariff calculation, counting stacks of wine barrels, ratio conversions in trade, and fortune-telling. These mathematical scenes not only reflect the daily life of the Song dynasty but also reveal the indispensable role of mathematics in that historical context. *The School of Athens* portrays a gathering of ancient philosophers, mathematicians, and scientists, symbolizing the revival of ancient Greek philosophy and culture during the European Renaissance. At its center stand Plato and Aristotle, positioned on the left and right respectively, corresponding to the mathematical and philosophical traditions represented by Pythagoras on the left and Euclid on the right. This study investigates Taiwanese college students' responses to the implicit mathematical narratives within these two historic paintings and examines their interpretations of the role mathematics played in the respective cultural contexts.

Conceiving a history of mathematics course for pre-service mathematics teacher education in Hungary

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In this presentation, I will discuss the development of a new history of mathematics course for prospective secondary school mathematics teachers in Hungary. I will first teach this course in the fall of 2026. First, I will talk about the context: the place of the course in the teacher education program on one hand and the situation of history of mathematics (especially the problem of available resources) on the other. Then I will talk about different questions and aspects I take into account for the design of the course (Fauvel & Maanen, 2002; Jankvist, 2009; Siu, 2007) as:

1. Diversity of different reasons to include history of mathematics in mathematics education and in teacher education
2. Diversity of different ways to include history of mathematics in mathematics education and in teacher education
3. Potential difficulties of including history of mathematics in ordinary classrooms
4. Diversity of mathematical domains (with attention to Hungarian school curricula and teacher education curricula)
5. Diversity of different cultures and historical periods
6. The available resources in Hungary (and especially in Hungarian)

Concerning this last point, as I explained in (Gosztonyi, 2014), there is only a limited number of resources in Hungarian. Hungarian mathematical research started quite late, with quite advanced mathematics in most cases, not accessible for ordinary teachers neither for students. Furthermore, only a limited number of these publications are in Hungarian. Concerning secondary resources, most of these are old (published 40 or 50 years ago) and of an encyclopædic nature. In my presentation, I will take a moment to show that the question of resources appears in many different countries and cultures concerning history in mathematics education, but in quite different ways (Fauvel & Maanen, 2002; Gosztonyi, 2014).

After discussing the problems related to resources, I will present some specific Hungarian cases and resources that could be relevant to include into my course:

1. The contribution of Hungarian scientific travellers to the circulation of knowledge in the 18th century;
2. The case of the Bolyai's and the emergence of hyperbolic geometry;
3. The importance of discrete mathematics in Hungarian mathematical research and education;
4. A "heuristic" epistemology of mathematics, developed in the 20th c. Hungarian context and its impact on mathematics education.

Finally, I will present the plan of my course, based on the preceding considerations. In the discussion, I will ask participants to share their own experiences and discuss aspects of local adaptations of history of mathematics courses in their own countries.

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Co-production of knowledge and training of indigenous teachers in the Amazon

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The initial and continuing training of indigenous teachers faces a structural challenge: effectively integrating traditional knowledge into academic curricula, especially in the field of Mathematics Education (Nery; Mendes, 2021; 2023). This tension highlights an epistemological impasse between hegemonic school models and the unique ways of knowing of indigenous communities, whose educational practices are based on orality, collectivity, and a symbiotic relationship with the territory. The core of the problem lies in the difficulty of recognizing and legitimizing ancestral knowledge as valid foundations for the construction of intercultural pedagogical proposals. The marginalization of this knowledge compromises the quality of indigenous school education and the epistemic autonomy of native peoples. In this context, the following question emerges: what is the role of co-production in the training of indigenous teachers for mathematics education, considering the sociocultural practices of the ethnic groups in Amapá and northern Pará? This communication aims to discuss the engagement of indigenous teachers in the co-production of activities on mathematics education, valuing the sociocultural diversity of the ethnic groups in Amapá and northern Pará. The investigation is linked to the research and extension activities of the Study, Research and Practice Group in Intercultural Education in Natural Sciences and Mathematics (GECIM), in conjunction with the Intercultural Indigenous Degree Course at the Federal University of Amapá, Binational Campus, in Oiapoque, on the border between Brazil and French Guiana. The methodology is based on Participatory Research (Borda, 1985; Brandão, 1985), the Systematization of Experiences (Jara, 2006; 2020), and the Theory of Objectification (Radford, 2018; 2021). This latter perspective conceives of Mathematics Education as a political, social, historical, and cultural endeavor aimed at the dialectical formation of reflective and ethical subjects who critically position themselves within historically and culturally constituted mathematical practices (Radford, 2018). The production of empirical data occurred through oral, written, and audiovisual records, generated in the interactions between the co-producers, who are indigenous teachers in the process of initial and continuing training. The choice of the term co-producers reflects the theoretical- methodological positioning that guides this study. According to Nery (2023), co-production refers to a social process of knowledge construction that incorporates community principles, values indigenous languages, promotes intercultural dialogue, and affirms the self- determination of native peoples. This perspective contributes to the articulation between school knowledge and traditional knowledge, strengthening the construction of contextualized, ethical, and plural educational practices.

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Developing Mathematical Skills: The Challenge of History of Mathematics in the Classroom

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The use and integration of the history of mathematics in the classroom has become increasingly expressive in the educational context. The recent reform of the mathematics curriculum in Portuguese secondary education ask for the use of notable episodes and problems from the history of mathematics. Different researchers highlight the potential benefits of using and integrating historical sources in maths teaching. To provide the humanization of the subject itself, to allow the development of a mathematical culture, to promote the deepening of concepts and procedures, and to enhance contact with different resolution strategies for the same problem, are some of the factors presented that support the relevance of its use and integration. However, one of the major challenges faced by integrating the history of mathematics in the classroom is that its use is not limited to addressing curiosities, facts, or historical events. The use and integration of the history of mathematics in the classroom goes beyond pointing out facts or presenting mere curiosities; it allows for the development of various mathematical skills, such as problem-solving, mathematical communication, mathematical reasoning, mathematical argumentation, the use of different representations, and the establishment of connections. This presentation aims to make known a set of tasks, grounded in the history of mathematics, which seeks to achieve ways to promote the use and integration of the history of mathematics in the classroom, aiming to develop these skills. These tasks were carry out at middle and secondary classes of Portuguese's educational context and were designed to provide students with different resolutions, enabling them to compare different strategies. Thus, some of the problem-solving processes produced by the students, or even reflections made by them, are presented, illustrating how they interpret and perform tasks within the history of mathematics. The resolutions of these tasks highlights that students use different representations, establish connections within mathematics itself, and develop mathematical argumentation and communication based on their reasoning processes or present by others.

Doing analytical geometry is not like playing a tune by turning a barrel-organ

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In his *Confessions*, Jean-Jacques Rousseau wrote that he did not like solving geometry problems using equations because it was like "playing a tune by simply turning the handle of a barrel-organ." Today, many students struggle with analytical geometry, accumulating "errors" as if they were trying to memorize mechanisms they do not understand. Following Hans Freudenthal, we seek to examine the historical gap that has developed over time between the invention and teaching of these mechanisms. To do this, we examined some forty French textbooks and reissues from the late 18th century to the late 19th century. We have identified four characteristics.

1. The authors begin by applying algebra to geometry to solve problems.
2. They adopted two opposing orders of presentation. The first authors followed history, moving from the concept of the equation of a curve to that of the coordinates of a point. Their successors proceeded with an "elementation," moving from the coordinates of a point to the equations of lines and then curves.
3. Several authors consider the obstacles related to negatives, continuity, and homogeneity.
4. Many authors combine algebra and geometry by associating curves with analytic expressions, as well as vice versa.

The aim of this return to history is to distinguish our teaching, to take a step back, and to present devices and problems inspired by history.

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Educational Change Mediated through Textbooks: A Comparative Analysis of 3+1 Greek Mathematics Textbooks

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Mathematics education underwent multiple stages of development during the 20th century. The postwar period has often been the focus of research, primarily due to the Modern Mathematics reform and the introduction of new content. However, significant changes were already occurring in the first half of the century.

This article examines the teaching of quadratic equations through a series of textbooks (I. Hatzidakis, N. Sakellariou, Chr. Barbastathis) that shaped Greek mathematics education from the late 19th century onwards, as they were widely used and authored by prominent members of the Greek mathematical community. Significantly, these textbooks were either uniquely approved by the Greek Ministry of Education, or at times that laws permitted multiple textbooks, were the most popular textbooks nationwide.

Particular attention is given to the textbook market, the institutional framework for approval and circulation, the commercial competition among publishers, and interpersonal rivalries, all of which played a crucial role in these developments. After analyzing the three main prewar textbooks, the article briefly addresses how the same topic was approached in the 1960s within the framework of the Modern Mathematics reform.

This contribution elaborates on the author's previous research on the history of Greek mathematics education, which primarily focused on the implementation of the Modern Mathematics reform in Greece, and draws connection lines with the shaping of the mathematics education landscape in Greece during the previous decades.

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Effect of hand games in enhancing the algebraic thinking skills of Native American high school students

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This study explores the impact of traditional Native American hand games on the development of algebraic thinking among middle school students, highlighting how culturally rooted, embodied practices can serve as powerful tools for mathematical learning. Drawing from frameworks in embodied cognition and culturally responsive pedagogy, the study examines how rhythmic patterns, strategic movement, and communal play inherent in hand games support students' understanding of algebraic concepts such as pattern generalization, variable manipulation, and functional relationships.

Through a mixed-methods approach involving classroom observations, student interviews, and pre/post assessments, the research reveals that integrating hand games into math instruction not only enhances conceptual understanding but also fosters engagement, confidence, and cultural affirmation. The study argues that hand games bridge informal knowledge and formal mathematics, offering educators a pathway to design inclusive, community-connected algebra curricula that honor Native American ways of knowing while deepening algebraic thinking.

Exploring the impact of historical mathematical contexts on critical thinking in secondary education

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The study examines how integrating historical contexts–such as the evolution of mathematical concepts, classic problems, and biographical narratives–into math activity scenarios can enhance critical thinking among middle and high school students. The research explores how, by displaying the "human side" of mathematics, we can promote conceptual understanding and encourage reflective inquiry.

The History of Mathematics is a recommended course for undergraduate and graduate programs in fundamental mathematics, as well as in mathematics education. The particularity of the curriculum for this course in programs related to mathematics education stems from the need to reflect in its content certain aspects of the evolution of didactic approaches in mathematics. The study highlights how such approaches support teachers in creating dialogic and cognitively stimulating learning environments. Preliminary findings suggest that historical contextualization not only deepens students' understanding of mathematical structures, but also cultivates habits of questioning, reasoning, and connecting ideas across disciplines.

The study proposes a pedagogical model that integrates elements of the history of mathematics in the design of curricular and extracurricular activities, with a particular focus on students who demonstrate high mathematical potential. The research also addresses the implications of such integration for differentiated instruction, especially in programs targeting gifted or high-achieving students. The model is designed to assess the practical skills of future mathematics teachers in designing activities that integrate elements of the history of mathematics, both during classes and during the internship. Practices show that the integration of elements of the history of mathematics in mathematics teaching, reflecting the epistemology of concepts and the ways of logically substantiating them, stimulates critical thinking and influences students' motivation to learn. This is a qualitative research, with elements of action research. The evaluation indicators target the areas critical thinking (logical reasoning, formulating questions, connecting ideas); motivation (active involvement, expressed interest, voluntary participation, consistent portfolio with accumulated materials); conceptual understanding (clarity in expressing mathematical ideas, correct use of terms); reflective capacity (the ability to analyse the learning process, to make interdisciplinary connections, to report on results in front of colleagues).

By promoting the idea of integration of historical mathematical knowledge in both initial and in-service teacher training, the study contributes to the development of inclusive and intellectually enriching mathematical instruction.

Exploring the Integration of Artificial Intelligence in a History of Mathematics for Educators Graduate Course

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This study examined how generative artificial intelligence (GenAI) tools were integrated into a graduate-level teacher education course, titled History of Mathematics for Educators, to support math teachers' engagement with the teaching and learning of the history of mathematics in K-20 math classrooms. The course was designed around the premise that history offers a powerful lens for understanding mathematics as a human, evolving endeavor, while GenAI can serve as a catalyst for broadening access to historical sources and deepening pedagogical reflection. The class comprised a diverse group of five students, which included four master's and one doctoral student. Three were international students, while one already held a Ph.D. in mathematics and taught in the university's mathematics department. The students brought varied professional backgrounds, including primary, secondary, and postsecondary teaching in both formal and informal contexts and professional tutoring.

GenAI was intentionally used by both instructor and students in intentional ways. As the instructor, I used various GenAI tools as "lesson planning partners." I prompted GenAI to generate multiple instructional suggestions for each class meeting, create discussion prompts and potential responses, and design reading check assessments with solutions. I also used GenAI to summarize chapters from the course's main texts, A Brief History of Mathematical Thought (Heaton, 2017) and The Crest of the Peacock: Non-European Roots of Mathematics (Joseph, 2010) as memory refreshers because it had been a while since I read them and taught the course. When GenAI suggested readings, I verified the authenticity and relevance of the sources. I was intentional in ensuring that GenAI was used as a reflective partner that supported instructional planning and intellectual curiosity.

I was transparent with my students about my intentional use of GenAI. We had a discussion during the first session at the beginning of the semester through a structured discussion on GenAI's educational implications, guided by a syllabus policy that clarified when, how, and for which tasks GenAI could be used. Students were asked to disclose any use of GenAI in their assignments and reflect on how it influenced their thinking. An example of a course assignment required students to use GenAI to design a lesson plan integrating a historical mathematical theme, such as infinity, Indian trigonometry, or the development of zero. Students documented their prompts, reflected on GenAI's influence on their pedagogical reasoning, and presented their lessons to peers.

To deepen this work, I also collected anonymous survey data from participants examining how they engaged with GenAI. Findings included:

- Accessibility and exploration: GenAI enabled more approachable engagement with complex historical content across diverse mathematical traditions, helping teachers explore underrepresented mathematical contributions and draw meaningful connections between global histories and their own instructional contexts.
- Pedagogical reflection: GenAI fostered meta-cognitive awareness of how historical narratives are interpreted, constructed, and represented in mathematics education, prompting teachers to reconsider whose voices and perspectives are elevated or omitted in traditional accounts.
- Critical stance: Teachers developed nuanced understandings of GenAI's affordances and ethical limitations, recognizing issues of bias, accuracy, authorship, and the importance of maintaining critical human judgment when incorporating AI-generated materials into their pedagogical design.

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Fagnano's problem and the epistemological role of geometry in eighteenth-century optimization problems

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The problem 'Given an acute triangle, how can we determine the inscribed triangle with the minimum perimeter?' was first published in 1775 by Giovanni Francesco Fagnano dei Toschi in Problemata quaedam ad methodum maximorum et minimorum spectantia. At first glance, it seems to naturally belong to the domain of differential calculus, which, by the second half of the eighteenth century, had already established itself as a powerful and widely used tool for solving optimisation problems. However, analysis of historical sources shows that Fagnano does not present an analytical solution, but rather explicitly defends the geometric approach.

This paper takes Fagnano's problem as part of the 18th-century debate on the tension between calculation and synthetic geometry. It offers a historical and epistemological analysis of the problem, demonstrating how studying historical sources can inform the integration of the history and epistemology of mathematics into mathematics education. By closely examining the structure, language and stated aims of Fagnano's text, we demonstrate that geometry is not merely employed as an auxiliary or illustrative method. Instead, geometry is deliberately promoted as a superior mode of reasoning, offering greater certainty, transparency and elegance than analytical procedures, which many mathematicians still perceived as technically effective but conceptually opaque.

In this context, comparing the analytical and geometric approaches historically provides an opportunity to question the various criteria of validity and significance that underpin mathematical activity. Elegance is not merely an aesthetic value here, but an epistemological criterion that informs demonstrative choices and enables the minimum to emerge as a necessary consequence of geometric relationships rather than the outcome of symbolic optimisation.

From a mathematics education perspective, Fagnano's problem provides a valuable case study for incorporating the history and epistemology of mathematics into education. This approach fosters reflection on the nature of mathematical reasoning and the variety of legitimate mathematical methods. Based on this historical-epistemological analysis, an activity was designed for pre-service teachers to encourage reflection on the epistemological implications of comparing the analytical and geometric approaches. H. A. Schwarz's proof, based on multiple reflections, was followed. This approach allows the minimum to emerge as a necessary consequence of spatial relations rather than as the result of a computational optimisation process.

The activity aimed to highlight the epistemological value of the geometric approach, as well as promoting conscious reflection on different ways of solving the same problem, and on the role of history in understanding mathematics.

Federigo Enriques' *Encyclopædia* Entries as an Epistemological Bridge and Didactic Resource for the Evolution of Fundamental Concepts: From Euclid to the Crisis of Foundations

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This study proposes a didactic approach grounded in Historical Epistemology (HeM), essential for presenting mathematical concepts in the classroom not in their final axiomatic form but through their dialectical and cultural development. The work identifies the *encyclopædia* entries authored by the renowned mathematician Federigo Enriques (1871-1946) – who contributed major entries to the *Enciclopedia Italiana* and served as editor of its mathematics section–as key resources for integrating Historical Epistemology (HeM) and the History of Mathematics (HoM) (Moyon, 2013). Their interdisciplinary nature (scientific and philosophical) (Furinghetti & Somaglia, 1998) enables the use of history not merely as contextual background but as a heuristic tool for the critical analysis of mathematical knowledge.

The epistemological analysis focuses on three entries (Enriques, 2019a, 2019b, 2019c), whose historical evolution marks fundamental milestones in mathematical thought:

- Point (Enriques, 2019a): The discussion examines the transition from its status as a primitive Euclidean entity (defined by negation) (Commandino, F. 1575; Euclide 1974; Proclo 1978) to Enriques' empirico-rationalist interpretation, and finally to its crisis as a model in modern physics (quantum mechanics), demonstrating the situational nature of the concept.
- Number (Enriques, 2019b): The discussion compares the Euclidean definition of natural number (Acerbi, 2007) with its subsequent evolution (Commandino, 1575; Busard, 1991; Clavius, 1602), ultimately highlighting the distinction between speculative arithmetic, as seen in Boethius (Guillaumin, 1995), and practical arithmetic, as

in the *Liber Abbaci* of Leonardo Pisano (Boncompagni, 1857), and the progressive autonomy of the concept from metaphysical assumptions.

- Proof (Enriques, 2019c): The focus is on the model of proof proposed by Enriques, which stands in clear opposition to the Euclidean ideal of "absolute truth." Proof is presented as a procedure of successive approximations– an instrument of discovery and a dialectical process in which error functions as a cognitive stimulus (in line with Lakatos' methodology) (Lakatos,1976) – and as fundamental for enabling students to construct profound mathematical meaning.

The study aims to show how the epistemological analysis of key foundational terms in mathematics enables educators to enhance the cultural dimension of the discipline (Barbin, 1997, 2022; Enriques 1906). Introducing the historical genesis and critical implications (Furinghetti & Radford 2002; Grugnetti et al. 2000) of these concepts transforms the learning environment into a laboratory of critical thinking, essential for fostering in students a deep understanding of the evolving nature of mathematical knowledge.

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For a classroom in-depth study of the knowledge of the number e and the exponential function

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In this communication, we present some key moments of the path proposed to a fifth-year class at the "Galileo Galilei" Technical Institute in Genoa (Italy), aiming to foster a conscious and shared understanding among students of the meaning of the number e and the exponential function.

This path also has the goal of ascertaining whether the intellectual enhancement of students' abstract vision in the mathematical field can improve their operational and modeling skills regarding real phenomena and situations.

The students start from the already acquired knowledge of the mathematical model for the discharge of a capacitor, which involves the use of an exponential function. The reflection begins by questioning the meaning they attribute to the number e and what "image" they have of it, for example, by comparing it to the geometric image they have of pi as the ratio between the circumference and its diameter. This reflection leads to the origins of the number e with references to readings from the works of John Napier (who used it as the base for logarithms), Jacob Bernoulli (who provided an initial theoretical definition of it as the limit of a sequence, for the practical purpose of calculating compound interest), and a very young Leonhard Euler (to whom we owe an early notation of ex within a formula indicating the tangent of an angle to describe cannon fire experiments, a notation still in use today).

Further in-depth studies followed, regarding the nature and characteristics of the exponential function as a mathematical object suitable for achieving mathematical analogy, understood as the method that consists of finding "affinities" between phenomena, even very different ones, at least one of which can be mathematically described. Once the analogies are established, this description can be considered as the mathematical model for the "similar" phenomena examined.

Starting from the modeling of a capacitor's discharge, students were asked to work on other situations involving the use of the exponential function. Historical readings supported the students' construction, under the supervision of the teachers, of mathematical models for population growth and radioactive decay.

To enhance students' perception of mathematical modeling techniques, the solution of certain problems was proposed with the aid of numerical analysis techniques.

From MathPods to PodsMath: The History of Mathematics as a Catalyst for Inclusive and Accessible Digital Learning

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This research describes the development of PodsMath, an innovative digital platform for teaching mathematics. The proposal integrates the History of Mathematics, Digital Technologies, and inclusive pedagogical practices, aiming to overcome learning barriers and promote the democratization of knowledge. The project arose from the growing relevance of educational inclusion in the Brazilian context, where mathematics is often seen as a complex discipline, and its teaching becomes even more challenging for students with visual and hearing impairments. The absence of adequate resources intensifies these difficulties and contributes to the educational exclusion of these students. In this scenario, digital technologies stand out as effective tools, offering alternative teaching formats and greater autonomy. Podcasts, in particular, have proven to be a promising educational resource capable of presenting complex content in an accessible and engaging way. PodsMath was conceived from the reformulation of a previous project, MathPods, which, despite being innovative, had limitations in its interface, such as accessibility flaws and low audio quality. The analysis of similar initiatives, such as Matematicast and Matemática Cantada, reinforced the need for a platform that combines organization, accessibility, and quality content. To address these issues, PodsMath was planned to go beyond simply providing audio. The platform includes podcasts with carefully crafted scripts that explore mathematics from a historical and cultural perspective, in addition to incorporating essential accessibility features. The integration of the History of Mathematics is not random but rather a strategy to re-signify school content, making it more contextualized and interesting. The link between this historical dimension and digital technologies allows for the exploration of narratives in an interactive and attractive way, promoting engagement and conceptual understanding. The main objective of the project is to develop and implement a digital platform that promotes the teaching of mathematics in a more inclusive, egalitarian, and attractive way.

The specific objectives include:

1. developing scripts and recordings in accessible language with high audio quality;
2. producing complementary materials, such as transcripts and descriptions, to ensure accessibility for audiences with hearing and visual impairments;
3. exploring mathematical concepts from a historical and cultural perspective;
4. promoting innovative learning experiences that integrate the history of mathematics and digital technologies.

Currently in its initial phase of development, PodsMath is expected to make significant contributions to Mathematics Education, especially regarding inclusion, methodological innovation, and the promotion of a more critical, contextualized, and accessible teaching approach, which aligns with contemporary discussions on the democratization of knowledge.

From Pascal to Leibniz: Intuition in (re)discovering Calculus

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This paper examines the historical development of calculus through the complementary yet contrasting approaches of Pascal and Leibniz, proposing their methodologies as a foundation for more intuitive pedagogical practices in contemporary mathematics education. By tracing the evolution from Pascal's geometric insights to Leibniz's algorithmic innovations, we demonstrate how understanding this historical progression can revolutionize calculus instruction.

The investigation begins by establishing the mathematical foundations laid by both scholars through their parallel developments in combinatorics–Pascal's triangular numbers and Leibniz's harmonic triangles–and their subsequent applications to infinite series summation. These early contributions reveal the complementary nature of their mathematical thinking and provide essential context for their later innovations in infinitesimal analysis.

The analysis then focuses on Pascal's groundbreaking geometric contributions, particularly his treatment of the cycloid and other mechanical curves. Pascal's 1658 publication under the pseudonym Dettonville presented these curves as exemplars of geometric comprehensibility, employing the method of indivisibles and establishing sophisticated relationships between curved and rectilinear magnitudes. These works not only solved previously intractable problems but also established a new standard for mathematical rigor and geometric intuition.

Building upon this foundation, the paper traces how Pascal's geometric approach profoundly influenced Leibniz's revolutionary development of calculus as a systematic algorithmic method. However, this transformation was not merely mathematical but philosophical, reflecting Leibniz's broader intellectual framework encompassing his search for a characteristica universalis, the principle of continuity, and the principle of sufficient reason. These philosophical underpinnings fundamentally distinguished Leibniz's approach from Pascal's purely geometric methodology, ultimately enabling the algorithmic power that defines modern calculus.

The paper concludes with a pedagogical proposition: that contemporary calculus instruction would benefit significantly from reintegrating the intuitive geometric foundations exemplified by Pascal with Leibniz's systematic infinitesimal methods. This historical approach offers students a more conceptually grounded understanding of calculus, moving beyond rote algorithmic manipulation to genuine mathematical comprehension. We argue that this pedagogy of intuition, rooted in the historical development of the discipline, can address persistent challenges in calculus education while honoring the profound insights of these mathematical pioneers.

From Research to Practice and Back: A Teacher Education Course on Integrating History of Mathematics

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Far beyond using the history of mathematics (HM) as a repository of dates, names, and curiosities, its integration with mathematics teaching can influence how contents are approached, how curricula are structured, and how students and educators conceive history, mathematics, and mathematics education.

With this understanding, our research group CHEMat (Collective on History in Mathematics Education) has been developing, over the past six years, a long-term project with three main goals: to map and describe historical insertions in mathematics textbooks; to promote critical analyses of the results; and to design didactic materials for a more meaningful integration of HM into teaching.

Although the number of historical insertions in Brazilian textbooks is considerable, this integration remains superficial. Generally, anecdotes and curiosities prevail over approaches that foster mathematical learning (Amadeo et al., 2025).

Hence, CHEMat began to develop initiatives focused on teacher education (e.g., Bernardes, Moustapha-Corrêa, Amadeo, 2025). This movement–from research to practice–led to the creation of a course designed to discuss possible pathways for integrating HM into mathematics teaching. Such a proposal resonates with a longstanding concern in the field about preparing teachers for this integration. In the Brazilian context, Cavalari et al. (2022) reinforce this challenge, noting that mathematics teacher education programs still dedicate little space to such discussions.

The authors work in a Mathematics Teacher Education Program that includes a mandatory HM course, whose syllabus does not address how to integrate history and teaching. Thus, we proposed an elective course structured around three units: analyses of historical insertions in mathematics textbooks; theoretical discussions on possibilities, difficulties, limitations, and benefits of integrating HM into teaching, based on literature in the field; and workshops featuring activities inspired by past mathematical practices and their historical, social, and cultural contexts.

The course was offered in the first semester of 2025, totaling 60 hours, with 21 enrolled students and no prerequisites. A general impression was that it encouraged students to rethink the role of history in teaching: from merely motivational to a more meaningful role for learning. They recognized the challenges of integration, as it requires planning and must "fit" within the curriculum, suggesting that the discussions fostered critical reflection.

The paper will outline how the CHEMat has moved from research findings to a design of a teacher education course. We intend to present some insights from its implementation and reflect on the possibility of moving in the opposite direction, from practice back to research. We expect to revise the proposal based on reflections from this first experience and develop a study aimed at offering this course to in-service teachers.

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Genèse du Calcul Vectoriel à l'École des Mines d'Ouro Preto (Brésil) au début du XX^e siècle

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L'École des Mines d'Ouro Preto (EMOP), fondée en 1876, représente un jalon dans l'histoire de l'enseignement supérieur brésilien par son engagement envers l'enseignement et à la recherche. Son importance est incontestable, couvrant non seulement l'enseignement et la production de connaissances dans ces domaines, mais aussi la recherche en mathématiques, comme l'étude des vecteurs, un sujet pertinent dans les sciences.

Traditionnellement, les études publiées indiquent que Theodoro Augusto Ramos (1895-1937) fut le pionnier de la systématisation du calcul vectoriel au Brésil, publiant en 1927 un ouvrage en portugais, puis en français, contribuant à la diffusion de ces connaissances. Cependant, des nouvelles sources historiques trouvées dans les archives de l'EMOP suggèrent que les vecteurs faisait déjà l'objet d'étude et d'enseignement dans l'institution depuis le début des années 1920. Notre étude se situe dans la Première République du Brésil (1889-1930), période marquée par une quête croissante de "l'identité brésilienne" et par une révision identitaire. Ce désir d'une culture propre, exprimé par le Mouvement Anthropophagique d'Oswald de Andrade qui proposait de "dévorer" l'influence étrangère pour générer quelque chose d'authentiquement national, et par la Semaine d'Art Moderne au début des années 1920, ne s'est pas limité à l'art. Dans les domains des mathématiques et aussi en science, Ramos et Santos illustrent cette tendance : bien qu'ils aient des références européennes, ils ont cherché à élaborer le contenu du calcul vectoriel avec une identité brésilienne, se distinguant des simples traductions d'ouvrages étrangers. Dans ce contexte, l'objectif est de présenter la genèse et le développement de la connaissance scientifique du calcul vectoriel au XX^e siècle à l'EMOP, en soulignant le rôle du "Collectif de Pensée" depuis sa genèse, son introduction et sa consolidation. Les premières indications de ce mouvement remontent à 1920, lorsque le professeur Geraldo Silveira a introduit des notions sur les vecteurs dans ses cours de mécanique générale, approfondissant le sujet au fil des années suivantes. Par la suite, en 1924, le professeur Miguel Rocha présenta une brève exposition sur le sujet dans le cours de calcul infinitésimal. Son initiative, qui incluait l'encouragement de collègues et la mise à disposition de livres importés, principalement français et italiens, fut cruciale pour la "Circulation de Pensée" au sein de l'EMOP. Renforçant cet intérêt collectif, en 1925, Gastão Gomes publia un livre de mécanique rationnelle avec des pages dédiées à la théorie des vecteurs. Cet effort conjoint des professeurs a formé un "Collectif de Pensée" qui a culminé avec la publication, par Christovam Colombo dos Santos (1890-1980), d'un ouvrage en 1927, se distinguant par son originalité sur la scène brésilienne. Le livre de Santos est issu de ses leçons données en dehors du cursus officiel à l'EMOP. De plus, l'action de Santos fut décisive pour la constitution d'un "Style de Pensée" dans l'institution de Minas Gerais. Son influence est reconnue et explicitement créditée dans des livres ultérieurs sur le Calcul Vectoriel, tels que ceux écrits par ses anciens étudiants : Altamiro Tibiriçà Dias (en 1953), Edmundo Menezes Dantas (en 1962) et Antônio Moreira Calaes (en 1973).

Histoire des mathématiques et patrimoine culturel local : une expérience de formation des enseignants à Mayotte

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3

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Questionner la vision uniforme et universelle de l'enseignement des mathématiques est une problématique d'actualité. Les recherches récentes en histoire des mathématiques et en ethnomathématique ont mis en évidence une construction lente, collective, plurilingue et pluriculturelle des savoirs, marquée par des circulations, des emprunts et des hybridations entre groupes humains. Cela a permis de repenser la production, la transmission, la réception et la transposition didactique des concepts et pratiques mathématiques en termes historiques, culturels, socioéconomiques et institutionnels.

En France, ces réflexions ont eu des effets significatifs sur les dernières réformes curriculaires. Depuis 2019, l'histoire des mathématiques est entrée officiellement dans le cursus des lycéens (15-18 ans), non comme une simple intention déclinée au sein des objectifs généraux, mais sous la forme de contenus précis à travailler en interaction avec chacun des thèmes mathématiques du programme. Au niveau des collèges (11-15 ans), la réforme en cours va encore plus loin en introduisant explicitement, dans chaque partie du curriculum, une rubrique intitulée "Mises en perspective historiques ou culturelles".

À Mayotte, département français de l'océan Indien, nous travaillons depuis cinq ans sur cette double contextualisation des savoirs, à la fois dans le temps avec l'histoire, et dans l'espace linguistique et culturel par la prise en compte du patrimoine local. Le dispositif de formation des futurs enseignants de mathématiques du second degré que nous avons construit s'efforce ainsi d'exploiter simultanément l'histoire des mathématiques et la culture des apprenants dans l'intention de donner un maximum de sens aux contenus enseignés. Notre ambition avouée est de rendre la discipline plus inclusive et réflexive.

Pour les enseignants concernés et leurs formateurs, ce modèle se révèle être un outil qui favorise une objectivation critique du savoir mathématique en le restituant dans sa dynamique humaine, culturelle et institutionnelle. De plus, cette approche facilite l'engagement dans la réalisation de la tâche et augmente l'estime de soi de l'apprenant, ce qui est important dans des territoires comme Mayotte, où subsistent de très fortes inégalités scolaires et sociales.

L'objet de la communication est de présenter notre dispositif de formation qui se décline sur les quatre semestres d'un master en alternance. À partir de travaux d'étudiants, de questionnaires d'évaluation, d'entretiens et de vidéos de classes, nous étudierons les effets de la formation sur les représentations et pratiques professionnelles des professeurs stagiaires, et nous nous interrogerons sur les outils didactiques adaptés à l'analyse de situations multicontextualisées d'enseignement et de formation.

Historical sequential movements (HSM) as a way of approaching mathematics in school

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2

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Research related to the uses of history in mathematics education has multiplied dramatically since the 1990s, giving rise to various principles, methods, and modalities of didactic approaches for mathematics teaching programs. One problem identified in these approaches is: how do the research results connect, or should they be connected, to the themes, content, and teaching units proposed in the curriculum and textbooks adopted by the school during teaching practice? In this work, I present reflections on the problem and suggest actions that may enable connections, complements, and adaptations of historical information in the mathematical approach within classroom practices. To concretize these discussions and analyses, I present methodological approaches related to the elaboration of historical sequences based on problematization and historical investigation, as a process of adapting the historical information of academic mathematics to the units of mathematics in education, as presented in curricular programs and textbooks, which are proposed to be addressed in the classroom at each school level. My reflections focus on understanding and explaining how the historical-conceptual development of the mathematics to be taught can compose a sequential historical movement (SMH) that includes the mathematical elaborations established throughout time and space in connection with the mathematics taught in schools today. This is an interpretative and comprehensive movement about the historical processes of mathematical creation, which currently requires teachers to adopt new exercises that incorporate the dynamics of creation into teaching, in order to compose and execute methodological teaching strategies that promote students' mathematical learning. In this regard, I believe that mathematics education should establish a triangular connection based on three pillars: 1) historical knowledge in a sequential organization situated within the concepts related to the mathematical topic to be addressed; 2) the knowledge indicated hierarchically in the teaching programs, with the necessary in-depth analysis as highlighted at each level and teaching level; and 3) the ways in which it is presented in the textbooks adopted by the school, at each level and teaching level.

History of Mathematics and Mathematics Education: Key Themes, Dialogues, and Provocations

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In this communication, we examine several historiographical lines of force in research and writing on the history of mathematics education. Recent scholarship argues that mathematics, beyond the transmission of technical knowledge, should be understood as a human activity rich in cultural and historical meanings. Many authors have underscored the importance of cultural history for seeing mathematics education as a cultural practice shaped by social, political, and economic contexts.

Building on this discussion, we explore the relationship between the history of mathematics and the history of mathematics education, highlighting both points of intersection and methodological distinctions between these fields. We also consider what is specific about research in the history of mathematics education when compared with other sciences – such as physics and astronomy – particularly given the peculiarities of the abstract nature of mathematics in contrast to experimental disciplines.

Starting from a provocation, we reflect on the epistemological and formative bases required for research in the history of mathematics education: is it truly essential for a researcher to possess a mathematical background in order to understand the evolution of this discipline in educational contexts?

The answer is not immediate. The advantages and limitations of a technical background in mathematics contrast with those of a humanities background. While mathematical knowledge can facilitate a more precise understanding of content, it may also confine analysis to the internal logic of the discipline, overlooking broader cultural and institutional influences. Conversely, training in the humanities supports a more contextualized and critical view, revealing how mathematics – beyond its technical content – is shaped by cultural and political practices. Our analysis seeks to move beyond the dichotomy between technique and culture, suggesting that the history of mathematics education is not merely a sequence of pedagogical changes but a complex manifestation of social interactions, institutional contexts, and discourses of power.

By examining these intersections, we aim to contribute to the ongoing historiographical debate and to propose a widened perspective on the history of mathematics education – one that acknowledges its multiple layers of meaning and influence while remaining sensitive to the specificities of mathematical knowledge.

History of mathematics in pre-service secondary education teacher training in Brazil and Spain. A comparative analysis and implications

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Clark (2019) points out various research approaches focused on the relationship between the history of mathematics and mathematics education. One of them relates to studying the role of the history of mathematics in teacher training. It is widely acknowledged that history of mathematics can play a very important role in supporting and enhancing the development of the different domains that constitute the required knowledge for teaching mathematics, both from the pedagogical and mathematical point of view (Arcavi, 2023). Thus, it is not surprising that, as Schubring et al. (2000) point out, history of mathematics has been a topic covered in teacher education programs since a long time ago. However, some works suggest that there is a lack of knowledge about history of mathematics both among pre-service and in-service teachers (Gazit, 2012). In this work we aim to give an overview of the presence of history of mathematics in the pre-service secondary education mathematics teacher training programs in Brazil and Spain. These countries have different models for secondary education mathematics teacher training. In Spain, it has two stages. First, candidates complete a bachelor's degree in Mathematics or a related field that provides strong mathematical knowledge. After that, they complete a one-year Master's Degree that includes courses on pedagogy, educational psychology, teaching methods, curriculum design, and classroom management, along with a teaching practicum in secondary school. On the other hand, in Brazil, teacher education takes place in a single stage at the undergraduate level, through a specific bachelor's degree called Licenciatura. Since the mid-twentieth century, the education of mathematics teachers for secondary education (or its equivalent – the final years of basic education and upper secondary education) has been organized separately from the bachelor's degree in Mathematics. This distinction aims to integrate, throughout the teacher education program, courses with pedagogical and practical components, such as foundations of education, teaching practices, and supervised curricular internship, as well as to promote a continuous dialogue with the field of Mathematics Education. After comparing the situation in both countries, we reflect about how the presence and role of history of mathematics can differ according to the different possible teacher training models.

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History of mathematics in teacher education Amsterdam University of Applied Sciences

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3

In the teacher training system in the Netherlands a so called national box of knowledge ("Landelijke Kennisbasis" in Dutch) prescribes what knowledge and skills mathematics teachers must have. History of mathematics (HoM) is a part of this box. In both the bachelor and master mathematics teacher programs at the Amsterdam University of Applied Sciences (AUAS) HoM is therefore part of the curricula. In our presentation, these curricula, design criteria for the courses and experiences of our students will be presented.

In the bachelor program for mathematics teachers at AUAS the mandatory course on HoM is completed with an oral examen. Both the content of the course as the form, content and student experiences with the oral exam will be presented.

In the master program we use the 4S-framework (Agterberg, 2021) in the mandatory course on HoM. This framework consists of four formats: speck, stamp, snippet, and story. The formats are used for describing the appearances of history of mathematics in curriculum materials. In the presentation these formats will be explained based on examples from Dutch textbooks.

In the course, students do assignments on how specks/stamps in secondary textbooks can be upgraded to snippets and to stories. The content of the course, assignments and examples of student work will also be shown.

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History of mathematics in the classroom: the risk of an incomplete pedagogy

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1

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As a secondary school teacher and a mathematics education researcher, I share with the HPM community the values that motivate the use of history in mathematics teaching and learning. In my classroom work, I have conducted various teaching experiences with history: some have yielded satisfactory results, others unsatisfactory. I have also collected questions and critical comments from some students: "Why do we do these things, while my friends at other schools don't?", "Math is calculation, not history", "Tell us what we need to know about the content of the historical document we're reading" etc. As a teacher, I have reflected on the failures: what were the reasons behind them? what personal characteristics of the students might have contributed to them? what kind of change in the teaching approach do they require?

Through this presentation, I wish to share my reflections on the possibility of failure in experiences conducted throughout the history of mathematics, but now from a research perspective, that is, with reference to general pedagogical foundations. This reflection then raises other questions, such as: what are the essential pedagogical aspects? how can they be transferred to classroom practice? in planning and reporting experiences, are some emphasized to the detriment of others?

I view teacher reflection and researchers' pedagogical analysis as manifestations of responsibility toward students: ethical aspects. Questions may also arise regarding the values that inspire educators' initiatives oriented toward teaching mathematics through history, compared with those of the students for whom these initiatives are intended: axiological aspects.

I consider the triad 1. Ontology, 2. Epistemology, 3. Ethics (and Axiology) as the fundamental structure of a pedagogy, that is: 1. Role of history in identifying the origin of mathematical concepts, their definition and development, 2. Knowledge of mathematical concepts, 3. Relational ethics referring to the actors in the classroom and to the mutual responsibility in the construction of mathematical knowledge (I have put the reference to Axiology in brackets because, in the mathematics classroom, I believe there are difficulties in a preliminary analysis of the values that may be specific to students; this, however, does not mean abandoning reflection on them, which I believe should remain as an act of responsibility towards students).

The papers in the Proceedings of HPM and ESU show a tendency to prioritize onto-epistemological aspects, to the detriment of what I have identified as ethical (and axiological). This prompted me to use the term "risk" in the title, as I believe that neglecting ethical aspects increases the likelihood of failure for an educational initiative based on the history of mathematics.

How Quido Vetter (1881-1960) made history of mathematics an inherent part of mathematics teaching and teacher training in Czechoslovakia?

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3

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In 1917, the mathematician Quido Vetter (1881-1960) habilitated with his work on the methodology in history of mathematics. He argued that history of mathematics was more than a mere collection of facts and called for applying methods from related relevant fields, such as literary and linguistic studies.

Since the emancipation of the faculty of science at Charles University in Prague in 1920, Vetter gave lectures on history of mathematics to students of mathematics. At that time, mathematics teachers were essentially mathematics students who passed the required state exams in mathematics and also an exam in pedagogy. History of mathematics lectures and seminars provided a place for reflection on mathematical thinking and were well attended throughout the 1920s and 1930s.

After WWII, Vetter, who had then reached retirement age, was asked to give lectures to teachers of mathematics, but in the 1950s, historical sketches were included in textbooks, where they were openly declared to be a part of propaganda, spreading the materialist worldview.

In my talk, I will give an overview of topics covered by Vetter in his lectures and discuss the possible rationale for his low impact on postwar historical-mathematical sketches, as exemplified in mathematics textbooks of the time.

How the History of Mathematics Can Support the Discovery of an Original Proof? The Case of the Law of Sines

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2

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This paper examines how the history of mathematics can actively support mathematical discovery through a case study centered on an original proof of the law of sines. Rather than treating history as a descriptive background, the study adopts a heuristic and epistemological perspective, showing how a deep engagement with ancient trigonometry–specifically chord- based trigonometry–can lead to new mathematical insights. The paper situates the law of sines within its historical development, from ancient Greek chord tables used in astronomy to the decisive shift toward sine-based trigonometry in the Indian and medieval Arabic traditions. It highlights how early trigonometric reasoning relied on arcs and chords, with tools such as Menelaus' theorem and cyclic quadrilaterals playing a central role in triangle resolution.

Building on this historical framework, the author raises a deliberately unconventional question: whether an implicit "law of chords," equivalent to the law of sines, existed in ancient trigonometry, before the use of sines. By revisiting classical geometric properties– particularly the relationship between inscribed and central angles–the paper reconstructs a simple yet global proof of the law of sines, expressed through both geometric and algebraic arguments.

In fact, we will show how a deep understanding and mastery of trigonometry, combined with an unconventional question, enabled us to discover an original proof of the law of sines. The proposed proof differs from traditional ones by its conceptual unity and accessibility, making it suitable for secondary mathematics education. The study concludes by emphasizing the heuristic, didactic, and epistemological value of historical inquiry as a genuine driver of mathematical discovery.

Iconographic approach for studying Apollonius' Conics

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2

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In this presentation, I aim to describe an approach to the study of Apollonius' Conics (Ver Eecke, 1924), based on a series of figurative, or rather, iconographic symbols, that I have created and named σχήματα. Through these, and with the help of GeoGebra, I seek to unfold Apollonius' propositions by paying close attention both to the general statement (πρότασις) and to the demonstration (απόδειξις).

Within this approach, it is also essential to make explicit the important role played by geometric constructions (κατασκευή) in the unfolding of Apollonius' propositions, constructions that are generally not stated explicitly in the Conics itself but can be found in Eutocius' commentary (Decorps-Foulquier & Federspiel, 2014).

This perspective allows us to emphasize how certain tools that may have been common practice for a mathematician contemporary with Apollonius might not have been so for a commentator such as Eutocius, and even less so for a modern mathematician.

The σχήματα are conceived as a tools for in-service and prospective teachers, with the aim of adopting a synchronic approach (Fried, 2007) to the study of Apollonius' Conics, in the spirit of the words expressed by Fried (2007, p. 217): "But examining Apollonius' text closely, trying to see it on its own terms?which is examining the text historically?students might begin to realize that their way of thinking about the conic sections is different from that of Apollonius. At that point, they would be in a position to realize that our way of thinking is really our own; that is, it belongs to us by virtue of our time, our culture, our particular synchronous plane of ideas, to use Saussure's term."

The importance of an approach to the study of geometry based on visual intuition can already be found in Byrne's book (Byrne, 1847), which has been a souce of educational inspiration. The idea of studying Euclid's Elements through a sequence of figures also appears in the work of Leonardo da Vinci. In The Unknown Leonardo (Reti & Bührer, 1974, p. 62), we read that Leonardo often did not transcribe the Latin text of the propositions but translated the theorems into a series of drawings reproduced in the book.

In my presentation, I will unfold Proposition 11 of Book I of Apollonius' Conics using the σχήματα , showing how this approach not only fosters visual intuition, by highlighting the tools and methods characteristic of Apollonius' geometric way of thinking, but also provides a means of focusing on the mechanisms of deductive reasoning (Duval, 1991). These mechanisms are represented schematically through figurative elements that preserve a logically consistent connection with the proposition itself, rather than relying on rhetorical formulation. Finally, I will present an example of how such σχήματα can be used in the classroom, through an activity inspired by Leonardo da Vinci's images.

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La place de l'histoire des mathématiques dans la formation des futurs enseignants du secondaire au Maroc

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3

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La formation des enseignants au Maroc a connu une réforme en 2018. Celle-ci a été restructurée en deux phases complémentaires : la première, d'une durée de trois ans, conduit à l'obtention d'une licence d'éducation au sein des établissements affiliés aux universités (écoles Normales Supérieures (ENS), écoles Supérieures d'éducation et de Formation (ESEF) ou faculté des sciences de l'éducation). La seconde correspond à une année de formation de qualification dispensée dans des institutions affiliées au ministère de l'éducation nationale (Centres Régionaux des Métiers de l'éducation et de la Formation (CRMEF)). L'une des nouveautés apportées par cette réforme est l'introduction d'un module d'histoire et d'épistémologie des sciences pour les filières scientifiques (Mathématiques, Physiques-chimie, Sciences de la vie et de la terre et Informatique). Cependant, au Maroc, l'histoire des sciences n'est pas tout à fait une nouveauté dans la formation des enseignants. Elle y était déjà présente, peut-être d'une manière volontaire et non systématique, au moins dès la fin des années 1980 comme en attestent les documents pédagogiques relatifs à la formation que nous avons pu consulter et les témoignages que nous avons pu recueillir auprès des enseignants qui avaient assuré ce cours à cette époque.

Dans cette communication, nous allons essayer de mettre en relief, en nous focalisant dans un premier temps sur les mathématiques, la place de l'histoire des sciences dans la formation des enseignants avant l'avènement des licences d'éducation en 2018. Pour cela, nous nous basons sur une analyse, d'une part, d'entretiens conduits auprès des formateurs responsables de ce cours à l'ENS de Marrakech et d'autre part, des résultats d'un questionnaire en ligne adressé aux enseignants en charge de l'enseignement de cette discipline dans les centres de formation à l'échelle nationale.

Les brevets d'invention de matériel pédagogique : un regard nouveau sur la réforme des mathématiques modernes (1950-1980)

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5

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Au cours du XX^e siècle, l'enseignement des mathématiques est soumis à des transformations qui affectent méthodes, savoirs et programmes dans le cadre de réformes plus structurelles de l'enseignement des mathématiques et de travaux naissants en didactique des mathématiques. L'un des moments les plus emblématiques de cette période fut appelée "réforme des mathématiques modernes" et se déroula, en France, dans les années 1950-1970. L'évocation de cette période est associée traditionnellement à un chapelet d'expressions – abstraction, structures, généralisation – qui évacue la dimension matérielle et manipulatoire de la réforme, en tout cas telle qu'imaginée par ses promoteurs dans les années 1950-1970.

L'objet de cette communication est de redonner la place qui est la sienne au matériel pédagogique pour l'enseignement primaire, produit en France pendant la période sous-jacente à la réforme, à l'aide de sources très peu exploitées par les historiens de l'enseignement : les brevets d'invention déposés à l'Institut national de la propriété industrielle (INPI). L'exposé vise à montrer que l'acculturation aux mathématiques modernes dont bénéficient les enseignants, préalablement aux changements de programmes qui s'opèrent dans les années 1970, est un terreau fertile à l'invention de dispositifs, dont tous ne trouveront pas un débouché commercial, mais qui témoignent d'une indéniable pénétration de l'esprit dans la réforme dans les milieux pédagogiques de l'immédiate après-guerre. La communication inclura aussi une dimension comparatiste avec la prise en compte de deux autres espaces géographiques et pédagogiques qui furent aussi soumis aux "New Maths" : l'Angleterre et les États-Unis.

Mathematical machines as historical artefacts: epistemological meanings and mathematical competencies in a laboratory approach to the parabola

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In mathematics education, artefacts are frequently introduced to support students' understanding of abstract concepts. When these artefacts are historically rooted, however, their educational potential extends beyond manipulation and visualization: they embody mathematical laws, epistemological assumptions, and historically situated forms of reasoning. This contribution investigates the role of mathematical machines as artefacts of the material culture of mathematics and analyzes their impact on students' construction of meaning and development of mathematical competencies in upper secondary education.

The study is based on a teaching experiment conducted in a third-year class of an Italian scientific high school, within a laboratory-based approach to the concept of parabola. The activity was designed to compare three different approaches to the same mathematical object: (1) the use of historical mathematical machines, namely the string parabolograph and Cavalieri's parabolograph; (2) a guided construction using dynamic geometry software; and (3) paper folding activities aimed at highlighting focal properties of the parabola. The underlying hypothesis was that different artefacts, embodying different epistemological perspectives, would support different processes of meaning construction.

Mathematical machines are historical artefacts originally designed to generate curves through constrained motion, incorporating mathematical definitions and properties within their very structure. According to a classical characterization, a mathematical machine "forces a point or a figure to move according to a mathematically determined law". From a historical perspective, these machines reflect a conception of geometry grounded in construction, movement, and necessity, which played a central role in the development of geometry between the sixteenth and seventeenth centuries.

During the laboratory activities, students first interacted with the parabolographs, physically generating the parabola as a locus produced by mechanical constraints. Subsequently, they explored the same curve through dynamic geometry constructions and paper folding. The comparison among these approaches revealed significant differences in terms of students' engagement and conceptual understanding. Data analysis showed that the manipulative approach based on historical machines was perceived by students as more effective and meaningful than the other methods. In particular, the machines supported a stronger awareness of the defining properties of the parabola and fostered a deeper understanding of the relationship between geometric definition, construction process, and resulting curve.

The analysis of students' activity was conducted using the KOM framework. Rather than serving as an assessment tool, the framework was used as an interpretative lens to identify which competencies were activated in each approach. The results suggest that interaction with mathematical machines particularly enhanced competencies related to representation, reasoning, and the use of mathematical tools, while also promoting reflective awareness of the role of artefacts in mathematical knowledge construction.

From the perspective of history in mathematics education, these findings highlight the specific educational value of artefacts that carry historical and epistemological meaning. Integrating mathematical machines into classroom practice allows students to engage with forms of mathematical rationality that are historically situated yet epistemologically powerful. The study suggests that the history of mathematics can be effectively conveyed not only through texts and narratives, but also through objects that embody mathematical ideas in action, offering valuable insights for both classroom teaching and teacher education.

Mathematics in Cultural Context: AI Reflections on Ethnomathematics

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Mathematics cannot be viewed as a single universal language, as its principles and foundations vary across cultures (Rosa & Orey, 2010). The process of producing mathematical ideas incorporates each cultural group's own forms of knowledge construction and unique interpretations (Rosa & Orey, 2006). Ethnomathematics is used to express the relationship between culture and mathematics (Rosa & Orey, 2013). Ethnomodeling, an extension of this approach, is the study of mathematical ideas and methods developed by cultural groups (Rosa & Orey, 2013). Models developed by cultural groups are symbolic systems organized according to their own internal logic (Leton et al., 2025). Recently, artificial intelligence tools, such as large language models (e.g., ChatGPT), have begun to be used as an innovative tool in designing culturally based math activities (Muthukrishna et al., 2025).

In this study, the "Mathematical Modelling Problem Generator (MMPG)" ChatGPT tool (Ünsal et al., 2026), previously developed by researchers, was used. This tool is a GPT tool trained according to Lin's (2024) elements using a 955-page mathematical modeling archive. The purpose of the tool is to generate mathematical modeling activities based on the desired mathematical topic and to provide solution suggestions. In the first phase, five mathematical modeling problems related to Turkish culture were created using MMPG. This study was designed as a qualitative document analysis. In this descriptive study, the CRMT Lesson Analysis Tool framework by Aguirre and Zavala (2013) was employed, which enables teachers to evaluate mathematics lessons in terms of cognitive, cultural, linguistic, and social justice dimensions. Six of the eight dimensions related to focus served as the basis for data analysis. In the second phase of the research, a session was conducted using the Microsoft Edge ChatGPT tool (an untrained GPT model), which had no prior training in modeling. This tool was then asked to create a mathematical modeling problem related to Turkish culture. The problem it created was examined in terms of its characteristics as a modeling problem and compared culturally with the problems generated by the trained GPT tool, MMPG, based on CRMT dimensions.

As a result, when the mathematical modeling problems generated by MMPG in the context of Turkish culture were examined, it was observed that, except for a single modeling problem, it generally met the dimensions other than the "use of critical knowledge/social justice" dimension, which did not directly address themes of social equality and justice. It was observed that the untrained ChatGPT did not fully reflect the characteristics of the modeling problem. Although it had a structure that developed mathematical analysis and ratio-proportion skills, it only utilized cultural elements at a contextual level. It did not incorporate dimensions of social justice or social awareness. The findings indicate that AI tools have significant potential for designing culturally sensitive modeling activities. It is recommended that problems developed in different cultural contexts be examined comparatively in the future.

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Meeting halfway: How can History of mathematics be a resource for mathematics education research?

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1

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Surveys on the connections between history of mathematics and mathematics education systematically mention, among the various potential roles of historical knowledge, that of being a resource for mathematics education research. This potential role is particularly deeply-rooted in the French research communities (both historical and didactical), with a long-running "conversation on the potential benefits of historical inputs for didactics (Artigue, 1990) and on the many challenges of a collaboration that should remain true to the differing epistemologies of both "fields (Barbin, 1997; Chorlay & de Hosson, 2016; Jankvist & Kjeldsen, 2025).

This talk bears on a current research project which aims to investigate the conditions for an actual and fruitful use of historical knowledge by maths education researchers. More specifically, this case-study focuses on the reception, by maths education researchers, of a survey on the history on the notion of function that I published recently (Chorlay, 2025). Although this survey can be seen as a "neutral" historical account, pinpointing key stages in the historical evolution of the function concept and discussing large extracts from original sources, I made several design choices which I hypothesised would allow this survey to be a resource for mathematics education research. Among such choices: (1) I selected historical episodes which shed light on mathematical aspects of the function concept which maths education researchers identify as challenging (e.g., the difference between "function" and "formula"), (2) I selected a short list of concepts which I think make sense in both research fields (in particular, a key concept of "image" of a discipline) and used them to shape the historical narrative in a way that – I surmised – resonates with the interests and concerns of maths education researchers.

Over the 25-26 academic year, a series of semi-structure interviews with researchers in mathematics education will be carried out to test these hypotheses.

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One way in which scholarly historical knowledge in complex analysis is nuanced in contemporary Mexican university contexts

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In this presentation, we will share some findings of a doctoral research project which was based on the premise that the mathematical knowledge presented in contemporary university settings regarding topics in complex analysis is the result of a process of didactic transposition, which modifies the ways in which this knowledge arose during the historical development of this branch of mathematics by converting it into teachable knowledge.

Five original works related to Cauchy's integral theorem were analyzed under the previous premise, revealing the following forms of mathematical knowledge production related to the proof of the theorem. One method of proving the theorem relies exclusively on algebraic symbolism. A second type of proof resorts to the use of both algebraic symbolism and figures, with the particularity that every figure used in the proof must be accompanied by its algebraic counterpart. Finally, some proofs use algebraic symbolism as a means of mathematical justification and incorporate figures without counterparts in algebraic symbolism as a means of argumentation. These results are detailed in Piña-Aguirre and Farfán (2023). By identifying similarities and differences between the three types of proofs recovered from history and the ways in which content is currently presented in contemporary complex analysis textbooks, a series of tasks were designed in order to evoke forms of mathematical knowledge production similar to those used by historical subjects. The way in which these tasks were designed can be found in Piña-Aguirre and Farfán (2024), while the textbook analysis is shown in Piña-Aguirre et al. (2024).

Particularly, in this presentation, we will show how three Mexican undergraduate mathematics students use figures, both with and without algebraic counterparts, to work with some tasks related to the concept of complex integration. These results build upon the findings reported by Piña-Aguirre and Farfán (2025) concerning a pilot study conducted with mathematics teachers. This enables us to identify how the three forms of mathematical knowledge production used by historical subjects manifest in contemporary teaching and learning university scenarios.

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Planck's 1900 paper as a resource for an interdisciplinary laboratory with secondary school students

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This contribution combines historical–epistemological analysis with empirical classroom practice, focusing on the interaction between mathematics, physics, and technology through Max Planck's original work on black-body radiation and an interdisciplinary laboratory implemented with secondary school students. The historical analysis is based on a close reading of Planck's paper *Über das Gesetz der Energieverteilung im Normalspectrum* (1900–1901) in its original German text, supported by comparison with an English version.

Particular attention is paid to Planck's mathematical reasoning and to the Physics implications in terms of modern Quantum theory. The analysis highlights the epistemological tension between continuous and discrete models and the role of mathematical structures in mediating between thermodynamics, electromagnetic theory, and experimental constraints. From this perspective, mathematics appears not merely as a formal language, but as a constitutive practice shaping scientific knowledge and the epistemological status of constants of nature.

Building on this historical framework, an interdisciplinary laboratory was designed and carried out with students of secondary school. In the laboratory, students estimated Planck's constant by measuring the current–voltage characteristics of a simple electrical circuit containing a light-emitting diode (LED). By identifying the threshold voltage associated with photon emission at known wavelengths, students established a relationship between electrical energy and radiation frequency, using linear modelling to obtain an experimental estimate of the Planck's constant.

The activity integrates Mathematics, Physics and Technology (electronic components and measurement instruments). The historical texts were used to frame classroom discussions on the meaning of quantisation, and the limits of experimental determination, drawing explicit parallels between Planck's original reasoning and students' experimental practices. Qualitative analysis of students' written reports and classroom observations suggests that the historical–epistemological perspective supported conceptual understanding and engagement. In particular, students showed increased awareness of the non-obvious nature of the energy–frequency relation and of the role played by mathematical modelling in the interdisciplinary teaching and learning.

Potential of Collaboration between History and Mathematics Teachers: Inter/Transdisciplinary approach

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At the ESU-8 conference, we emphasized the rationale behind our research. Specifically, we discussed the potential and importance of collaboration between history teachers and mathematics teachers in the project of incorporating the history of mathematics into the classroom (Affan & Fried, 2019). We also examined the work of the Muslim mathematician and astronomer Abū'l-Wafā Buzj'ani (940-998), focusing on a course centered around his book *On the Geometric Constructions Necessary for the Artisan*.

Later, at ESU-9, we discussed the exact structure of the workshops with ten teachers (five mathematics teachers and five history teachers) who had participated in a course on the history of Islamic mathematics. We presented empirical results, which identified categories based on the characteristics of collaboration between the two groups (Affan & Fried, 2025). Our study, overall, aims to bring together history teachers and mathematics teachers to explore, firstly, what considerations and presuppositions they have when they approach the history of mathematics. Secondly, we investigate whether they can work together to produce a chapter on the history of mathematics for mathematics classes, and possibly for history classes as well. This suggests that integrating the history of mathematics into mathematics education would benefit from the interaction between these communities.

Such integration has been attempted to some extent before (Moyon, 2013). However, our approach is informed by the awareness that this attempt may face challenges similar to those it seeks to address–the tension between the beliefs and fundamental assumptions of history and mathematics teachers (Fried, 2001). Nonetheless, this awareness, combined with a careful selection of historical material, may enable the development of a genuine cooperative community. In this way, the solution to Fried's dilemma could involve viewing the history of mathematics as a fundamentally multidisciplinary field that necessitates expanding the community involved.

For this current conference (ESU-10), we will present a follow-up study focusing specifically on how this collaboration is manifested in the construction and subsequent transfer of a joint learning unit. We will detail the findings from this phase of cooperation, highlighting the various patterns of collaboration that emerged. Additionally, we will discuss the challenges of this interdisciplinary work and propose strategies to address them. Finally, we will offer recommendations for a new, collaborative approach to integration, with particular emphasis on its potential inclusion in mathematics classes.

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Problèmes inspirés par l'histoire des mathématiques : la valeur formatrice de l'analyse des sources historiques d'hier pour comprendre comment pensent les élèves d'aujourd'hui

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Certains auteurs (Arcavi & Isoda 2007; Arcavi, Drijvers & Stacey 2017) ont souligné l'importance de l'analyse des sources historiques pour développer chez les enseignants de mathématiques la capacité d'adopter le "point de vue de l'autre" afin de comprendre les raisonnements des élèves. Dans quelle mesure certains types de problèmes, que nous qualifions "inspirés de l'histoire", peuvent-ils nous aider à comprendre comment les élèves mobilisent leurs connaissances en matière de proportionnalité et d'opérations sur les inconnues, au-delà de leur niveau de maîtrise du langage algébrique? Comment certaines contextualisations favorisent-elles ou entravent-elles une approche de la résolution de problèmes qui aboutit à une solution lorsqu'une "équation" n'est pas strictement nécessaire?

Ces questions guident cette présentation, dans laquelle nous décrirons la construction d'une série de problèmes à partir d'énoncés et de procédures documentés dans des papyrus égyptiens et des tablettes d'argile babyloniennes. Notre objectif est d'analyser les conceptions des élèves en fin de primaire et en début de secondaire concernant la résolution de problèmes impliquant des inconnues. Nous formulons l'hypothèse que l'expérience préalable du travail algébrique à l'école influence la diversité des stratégies de résolution de problèmes. L'analyse ultérieure des travaux d'élèves révèle, entre autres, les aspects suivants: face à des problèmes de proportionnalité présentés sous forme de tableaux de données numériques, la plupart des élèves ont eu recours à la notation numérique, les résolvant avec une relative efficacité même lorsqu'il s'agissait de nombres non entiers. Comparée aux méthodes de résolution de problèmes égyptiennes, notamment la méthode de la fausse position, l'analyse montre que les élèves du primaire et du secondaire partagent la même approche par essais et erreurs, mais l'utilisation moderne des fractions et des expressions décimales leur permet d'aborder ces problèmes sans difficulté majeure. L'analyse des solutions des élèves du secondaire révèle que la modélisation et la manipulation algébriques modifient leur approche du problème et semblent exclure la méthode par essais et erreurs. Lorsque les problèmes acquièrent une difficulté relative – comme c'est le cas pour les problèmes babyloniens impliquant deux quantités inconnues liées linéairement, – les difficultés identifiées au primaire apparaissent : la complexité de la manipulation des schémas géométriques analytiques et le recours à la méthode par essais et erreurs pour trouver les résultats. Bien que notre intention ait été de promouvoir l'utilisation de représentations géométriques dans la conception des problèmes, d'autres solutions sont apparues: par exemple, certains élèves ont utilisé des triangles isocèles, même si cette caractéristique ne correspond pas aux consignes. Nous supposons que, outre la relative complexité de l'approche algébrique, l'existence de deux inconnues a introduit une complexité intrinsèque au problème, incitant les élèves à explorer d'autres méthodes de résolution. Il est donc nécessaire d'analyser les conditions de la pratique géométrique en classe, où elle semble jouer un rôle essentiellement iconique.

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Propositions to Sharpen Young Minds from York to Singapore

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The *Propositiones ad Acuendos Juvenes* represents a milestone in the history of mathematics and its teaching. It has benefited from the attention of historians of mathematics for over a century, but exists in English translation only since the 1990s, namely through the translation published in 1992 in *The Mathematical Gazette* by John Hadley and David Singmaster. The work is a collection of recreational problems, with solutions, including numerical and logical problems, and is commonly attributed to Alcuin of York (c. 735-804).

Alcuin was a deacon, poet, and teacher from York, in Northumbria – one of the Anglo-Saxon kingdoms that later contributed to the formation of present-day England. The text was intended for use in the context of the educational reform promoted by Charlemagne, in which Alcuin played a central role at the king's behest. Invited to the Frankish court around 782, Alcuin became the leading master of the Palace School and an intellectual advisor to Charlemagne.

The *Propositiones ad Acuendos Juvenes* is the first known collection of recreational mathematics problems, and thus marks the beginning the a tradition of using recreational problems in educational contexts, which includes, for instance, the commercial arithmetics of the late Middle Ages and the Renaissance.

In keeping with this trend, recently, this collection started to be scrutinized for inclusion in classrooms in some languages. Its broad encompassing of problems relevant both historically and culturally offer a unique learning opportunity both in problem solving methodologies as well as interdisciplinary projects. As many of them are problems that require an ad hoc reasoning, they manage to escape the traditional algebraic approach.

Here we propose to look at a few families of problems present in this historical treatise, offer their classic solution methods and propose new forms of solution that use the Singapore bar model method. This is to be used in today's classrooms, in an exploration involving both history and the most recent pedagogy of mathematics.

The choice of the Singapore bar model is motivated by its well-documented effectiveness in supporting conceptual understanding and problem solving. Singapore has consistently achieved outstanding results in international comparative studies such as TIMSS, which has drawn worldwide attention to its mathematics education framework. Central to this framework, the bar model provides a powerful visual and structural representation that bridges arithmetic and algebraic reasoning, making it particularly suitable for interpreting and reworking historical problems. Its adoption in curricula across numerous countries attests to its pedagogical value and its relevance for contemporary classrooms.

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Recreational mathematics in technical education during modern mathematics reform

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2

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Folha Informativa dos Professores do 1.º Grupo (E. T. P.) was a Portuguese periodical, published monthly between January 1967 and March 1972, aimed at Mathematics teachers in Technical Education. During this period, 66 issues and 9 supplements were published. Each issue consisted of a set of A4 sheets (between 5 and 37 pages), numbered and stencilled on one side, printed at the Escola Industrial e Comercial de Sintra, in Cacém, near Lisbon.

From the opening article of the first *Folha Informativa*, we conclude that it stemmed from a resolution of a colloquium held at the Escola Marquês de Pombal, in Lisbon, between December 16 and 22, 1966, intending to maintain a network of communication and sharing of ideas with some regularity, at a time when the reform of modern mathematics was being implemented in Portuguese technical schools. The first editions were organised into sections, one of which was "Recreational Mathematics" (MR – Matemática Recreativa), initially directed by Adriano Joaquim Vaz Velho Júnior, a graduate in Mathematical Sciences from the Faculty of Sciences of the University of Lisbon.

Recreational mathematics plays a fundamental role in the teaching and learning of mathematics in non-formal contexts, enabling knowledge-building experiences that value curiosity, experimentation, and the joy of learning. By integrating recreational mathematical problems with logical thinking, these activities promote the development of cognitive and social skills, fostering a more meaningful, contextually grounded understanding of mathematical concepts. This article aims to demonstrate the topics covered in this section during the journal's publication, in light of its historical and social context.

Methodologically, this study relies on a qualitative approach, grounded in the detailed examination of *Folha Informativa*, supported by official documents and other materials that help contextualise the social and cultural environment of the period.

This work demonstrates how recreational mathematics problems were integrated into a journal designed to support technical school teachers in implementing the topics of the modern mathematics reform, including its associated formalism and structures. We can interpret this to mean that technical school teachers, even during this period of reform, demonstrated a pedagogical interest in teaching and learning mathematics informally, beyond the classroom. The problems presented in this section of *Folha Informativa* are very current and well-suited for use in contemporary mathematics classes or in school mathematics clubs.

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Reviewing Descartes' construction of curves by continuous movement with secondary teachers, using a DGS

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1. In Descartes' *Géométrie*, a curve or a line, is constructible if it is possible to draw it through continuous movement and/or through certain articulated mechanisms, where the points on the line are obtained from the intersection of other lines, that could be curves, or even straight lines:

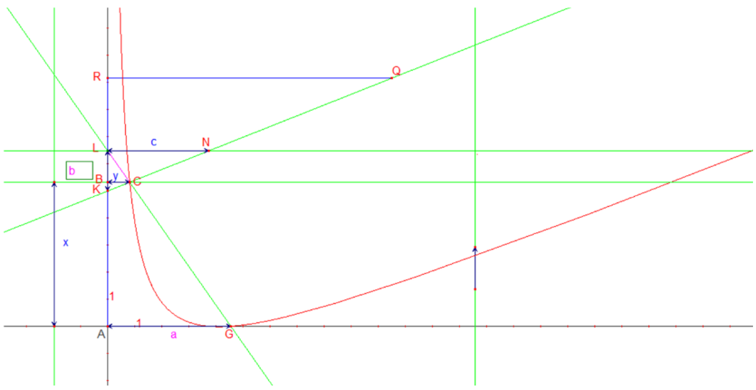
   [Descartes] has in mind the justification of a determining procedure in his *Géométrie*: the recourse to lines generated by moving intersection... As in a deductive chain, no matter how long it is, it can lead to a condition exact provided that the rules of the method have been respected, thus the generation of a curved line can be extremely composite provided that the rules of composition are respected. These rules refer in fact to only one: that the movement that leads from one curve to the other is continuous and entirely determined... By this means one can always have exact knowledge of its measurement. The measurement here should not be taken as a numerical notion but as a constructible magnitude, as it has been presented at the beginning of Book I. (Jullien, 1996, pp. 77-78, my translation)

   Descartes described an articulated system in motion: "These are 'moving planes' where the principle is the following: a starting line will move [in a parallel way] along an axis, and its intersection with an articulated ruler will produce a 'more composed line'..." (Jullien, 1996, p.83).

   Though it was general, the combinations of movements that Descartes had in mind (Bos, 1981, p. 310) involved only straight lines as moving parts, making it possible to generate curves in the plane, in particular, conics. For example, Descartes described the following construction artifact:

   A ruler GL pivots at G. It is linked at L with a device NKL which can be moved along the vertical axis while the direction of the line KN is kept constant. When L is moved along the vertical axis, the ruler turns around G and the line KN is moved downwards remaining parallel to itself. The intersection C of these two moving straight lines describes the curve GCE. (Bos, 1981, p. 311)

2. Using a DGS, it's possible to simulate the construction of the articulated system described by Descartes in his *Géométrie*. Furthermore, the construction of this kind of artifact makes transparent the geometric properties that remain invariant when the device moves vertically along the vertical (y-axis), which, in turn, is reflected in the establishment of the proportionality of the sides in the (similar) triangles that are/remained invariant under movement. This exemplifies Rabardel's concept (2011) of artifact operational transparency (Figure 1: Simulation of the artifact using DGS).



3. After leading a two-hour workshop on this version of Descartes' construction of curves (i.e., simulating the moving device using DGS) with secondary teachers, I asked them about the possibility of implementing it in their classrooms (for complementing the study of conics). What teachers considered as feasible in their classrooms, was that the students would be interested in studying or reading Descartes' original source.

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Seventeenth-Century Contributions to the Historical Development of the Method of Separation of Variables in Differential Equations

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The present study addresses the historical development of the method of separation of variables for solving differential equations, based on the examination of primary sources written by Gottfried Wilhelm von Leibniz (1646–1716) and Christiaan Huygens (1629-1695). Among these sources, four letters and two articles published in the journal *Acta Eruditorum* stand out – a scientific periodical published in Leipzig between 1682 and 1782. In this context, the initial delimitation of the research focuses on the 1684 volume of *Acta Eruditorum*, in which Leibniz published an article dealing with tangents, maxima and minima; however, on its first page the author also introduces the term *differentiali aequatione*, wxhich can be translated into Portuguese as *equação diferencial* (differential equation), as recorded by Struik (1959).

Based on this presentation, the objective of the study is to examine historical documents that address the historical development of differential equations, as well as to analyze some of the discussions reported in these documents. The methodology adopted is qualitative in nature, with a descriptive and documentary approach. For this purpose, documents from primary sources containing records of dialogues and publications were catalogued, making it possible to understand the path followed by Leibniz and Huygens in the development of differential equations (Nascimento, 2024). Regarding the textual investigation, a content analysis was carried out following the framework proposed by Bardin (2016), with the aim of examining the information extracted from the primary sources, in addition to providing comments on the historical context of the seventeenth century. In this process, scholars responsible for such works during the historical period under analysis were also identified, as well as the problematics associated with them. The results of this research show that the infinitesimal calculus disseminated in the seventeenth century constituted a fundamental element that enabled observations of the natural world to be modeled through differential equations and subsequently solved. In this sense, we demonstrate that problems involving mathematical observations of subtangent curves, as mentioned by Leibniz in one of his letters (Huygens, 1905), correspond to solutions of differential equations historically associated with procedures equivalent to what is now known as the method of separation of variables. Furthermore, Leibniz states that such curves may be as numerous as desired. This idea is corroborated in contemporary mathematics, since the solution of an ordinary differential equation may form a family of functions, which, in the 1684 article, Leibniz referred to as curves. The mathematical approaches of the seventeenth century differ significantly from those employed today, since the formal definition of function had not yet been established within the scientific community of that period. The analysis allows for a discussion of the epistemological constitution of solutions to differential equations prior to the formalization of the modern concept of function. These discussions reveal that the method of separation of variables emerged as a historically situated mathematical practice, whose understanding fosters epistemological reflections relevant to mathematics education.

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Shaping Czech Engineering Mathematics in the early 20th century: The Čuřík–Lerch Conflict

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The Industrial Revolution of the nineteenth century significantly increased the demand for mathematical education for a large cohorts of students, initially in civil engineering and later across mechanical, electrical, mining, metallurgical, and chemical disciplines. This development fundamentally changed the role of mathematics: it ceased to be viewed solely as the "queen of sciences" and increasingly became also a practical tool for mastering industrial production and achieving economic profit.

In the Czech lands, the rapid growth of industrial production in the second half of the nineteenth century contributed to economic emancipation and rising national self-confidence. This transformation was reflected in the development of Czech education at all levels–primary, secondary, and eventually technical–culminating in the expansion of the Czech Technical University in Prague, the growth of the Mining University in Příbram, and the founding of the second Czech Technical University in Brno in 1899.

Technical education, including mathematics and geometry, became a key factor in economic success. Traditional methods of teaching mathematics reached their capacity limits and no longer met the needs of engineering practice. Lecture-based instruction and individual examinations proved inefficient, creating demand for new textbooks explicitly designed for engineers and grounded in practical applications rather than abstract theory.

Following developments in Western Europe, where specialized mathematics textbooks for engineers appeared by the late nineteenth century, similar efforts emerged in the Czech lands. The first calculus textbook written in Czech for technical students was authored shortly before the First World War by František Čuřík, a trained mechanical engineer and professor of mathematics at an industrial secondary school, as well as a substitute lecturer at the Czech Technical University in Prague. Although his Foundations of Higher Mathematics are dated 1915, wartime conditions delayed its actual publication until 1916.

The reception of the textbook was deeply divided. Engineers and practitioners praised its style, visual approach, extensive practical examples, and original historical quotations illuminating the development of mathematics. In contrast, theoretical mathematicians criticized the work harshly. In the Journal for the Cultivation of Mathematics, published by the local professional organization (The Union of Czech Mathematicians and Physicists), Professor Mathias Lerch–one of the few internationally renowned Czech mathematicians of the period–published an uncompromising and polemical review, written in elevated style but marked by a one-sided view and personally offensive tone.

A defense of Čuřík's work was soon mounted by Vladimír List, Lerch's colleague at the Brno Polytechnic. He placed the textbook within the broader European context of engineering mathematics education, reiterated its merits, and explained the extraordinary wartime circumstances under which it was produced. Many of the criticized errors were corrected in the second edition published under much luckier circumstances in 1923, by which time Čuřík was a full professor at the Mining University in Příbram.

Although the episode ended well for Čuřík, debates over the proper conception of mathematics education at technical universities have continued in the Czech context to this day.

Spatial Thinking and Geometry in the Colombian Mathematics Curriculum: A History of the Present

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Curricular studies, understood as a history of the present within the educational sciences, analyze the norms of reason that shape expectations, discourses, and school practices (Popkewitz, 2010). Scientific disciplines such as physics, biology, and mathematics operate through specific epistemic frameworks, routines, and cultural and social arrangements that regulate knowledge production. In school contexts, curricula intersect with educational psychologies as mechanisms of translation that reconfigure these disciplinary domains into recognizable and bounded territories aligned with the logic of schooling (Popkewitz, 2020).

In recent decades, research in STEM (Science, Technology, Engineering, and Mathematics) and spatial thinking has recommended incorporating spatial thinking into science and mathematics curricula. Historical and empirical evidence shows that spatial thinking has been a distinctive feature of scientific and mathematical practices over time (Newcombe, 2017). In Europe, this incorporation has advanced within basic education, enabling the systematic integration of spatial thinking into curricula, the use of digital technologies, and the promotion of hands-on activities. However, challenges persist, including limited instructional time, insufficient attention to student diversity, and the pedagogical complexity of spatial teaching in mathematics and science classrooms (Bufasi et al., 2024).

From a curriculum perspective, implementing spatial thinking in America (Ibero-America, United States and Canada) requires recognizing its historical connection to the mathematics curriculum. Goemetry is a well-established field within mathematics education, with a long-standing concern for the role of spatial representation and spatial reasoning in teaching and learning processes (see Arcavi, 2003). Addressing spatial thinking from a historical standpoint is therefore essential. The purpose of this oral presentation is to map the continuities and discontinuities in the mathematics curriculum–through a historicizing of the present–that have shaped spatial thinking and the representation of space in the teaching of geometry. This study focuses on the mathematics curriculum of Colombia for four reasons. First, geometry has historically occupied a central place in the Colombian curriculum and has generated persistent tensions regarding its teaching (Vasco, 1991). Second, public policies have promoted a balance between national traditions and international curricular trends (Guacaneme-Suarez, 2025). Third, since 1998, the national curriculum has explicitly incorporated spatial thinking in relation to geometric systems (Ministry of National Education of Colombia, 1998).

Fourth, teachers have traditionally exercised autonomy in determining what to teach, guided by curriculum standards and policy recommendations (Guacaneme-Suarez, 2025).

The primary historical source for this study is the Colombian Mathematics Curriculum Guidelines published in 1998. Additional sources include earlier educational reforms, scholarly literature on spatial thinking, technical documents for curriculum design, and instructional materials. Preliminary findings from this ongoing PhD research highlight how spatial thinking and geometry are articulated through references to psychology within the curriculum framework. The school curriculum emphasizes active geometry, combining ideas about the central role of spatial thinking in scientific development with perspectives from cognitive psychology. At the same time, the national curriculum establishes a distinction between spatial thinking-geometric systems, and metric thinking-measurement systems. This distinction, however, tends to be diluted in later technical documents, where these domains are often treated in an integrated but less conceptually differentiated manner.

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Teachers' personal collections and the Math Education: the case of Rio de Janeiro in the second half of the 20th century

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This work aims to present a part of research, initiated in 2024 and still ongoing, concerning the methodologies for teaching mathematics used during the second half of the 20th century in the city of Rio de Janeiro, as well as their theoretical and methodological sources, resulting from the analysis of the personal collection of teacher Lúcia Maria Aversa Villela (called here APLMAV) from 1970 to 1995. The APLMAV was donated to the Center for Scientific and Pedagogical Documentation of Mathematics Teaching (CEMAT), located within the GHEMAT-Brazil in the city of Santos, São Paulo State, and consists of a collection of 440 books dated from 1936 to 2012, newspaper clippings, handwritten lesson plans, mathematics activities, and lecture notes. Thus, the bibliographic works were chosen as sources for the first phase of this study, and they underwent cataloging and categorization. Hence, the following cataloging criteria were defined: (i) Date of publication and/or re-publication; (ii) Publisher; (iii) Number of pages; (iv) Author; and (v) Language. The categorization was defined a priori by the following criteria: (i) Thematic focus of the work based on its content; (ii) Existence of an application manual for the teacher; (iii) National work or translation.

In this way, five categories were obtained in the first phase (partial results of the first phase), which are: 1. Works directly on mathematics teaching methodologies; 2. Works that do not directly address mathematics teaching methodologies, but have only one chapter; 3. General themes in Education; 4. Specific topics in Mathematics; and 5. General works. Based on the categorization, analyses were carried out using the hermeneutic and historical-cultural perspectives of the works in each category. For the selected excerpt, it was observed that some works in category 1 contained annotations in the "margins" relating to the teacher's lesson plans, assessments, and other information used in her work.

From this, it can be inferred that: 1. The teacher used her books as her own draft for the elaboration of her lesson plans and activity proposals; 2. Some records reflect some of the methodologies present in her pedagogical work, which are chosen systematically, as well as based on her own experience; and 3. The work is her main reference, being used sometimes as a methodological support, sometimes as a study/teaching resource.

Teaching mathematics in the 17th century. Pierre Hérigone's *Mathematical Course*

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One of the characteristics of the seventeenth-century mathematics is the progressive acceptance of algebraic procedures as a method for solving both arithmetic and geometric problems. In this context, Pierre Hérigone (1580–1643) wrote a Mathematical Course in six volumes. The first four volumes were published in 1634; the fifth in 1637; and the sixth, a supplement, in 1642. One of the most significant innovations of the Course is the creation of a new proof method based on a syllogistic procedure that employs a completely symbolic language, original to Hérigone, through which he developed its mathematical contents. This new logical-symbolic method enabled him to explain both classical and modern mathematics–pure and mixed–following a pedagogical project founded on three pillars: clarity, brevity, and rigor. In this way, he established a distinctive style that runs throughout his work.

In his Algebra, Hérigone presented, through his new logical-symbolic method, the analytical procedures that François Viète (1540–1603) had outlined and planned in his work *In Artem Analyticem Isagoge* (1591), developing them in his own manner, improving them in some instances, and applying them to new problems. For Hérigone, the analytical method, when applied to algebraic procedures, becomes a heuristic method for the discovery and demonstration of new results. Thus, he viewed and conveyed algebra as a tool capable of yielding universal results–not limited to any particular type of problem–and equally useful both for inventing and proving all kinds of theorems as well as for finding solutions to problems. He included most of the mathematical contents of his time, which led him to maintain a continuous concern regarding the way the sciences should be taught–a pedagogical imprint that shapes all the contents of the Course and which is essential to specify in order to better understand his work.

We will show the different pedagogical lines of Hérigone's work through the study of the prefaces to the various volumes of the Course, the contents he developed within it, and the analysis of several of his propositions. Our study will reveal how the perfect tool for carrying out his pedagogical project was his new logical-symbolic method for conducting proofs, and how his ongoing concern with the teaching of mathematics led him to develop new procedures and even new results.

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Teaching mathematics with machines: reconstructing Poleni's pedagogical activity via his lecture notes

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Giovanni Poleni (1683-1761) is associated with the cabinet of physical instruments he assembled in Padua throughout his life and used for teaching and research in experimental philosophy. Using machines to demonstrate physical phenomena or perform experiments was a hallmark of 18th-century science and a means of making physics accessible to the wider public. Poleni participated in this movement, introducing machines into his teaching of experimental philosophy at the University of Padua from 1739 onward. He believed that instruments provided visual illustrations of physical laws and obtained a classroom adapted for demonstrations, claiming that "experiments are meant to be seen." Were instruments in the 18th century limited to the teaching of experimental philosophy, or did they become part of the didactical tools of mathematics as well? In this oral presentation, we explore whether a similar revolution in teaching occurred in mathematics, especially in geometry, which Poleni taught from 1720 to 1761. Valuable evidence about teaching practices emerges from Poleni's extant lecture notes and related documents, allowing an overview of mathematics instruction at the University of Padua during the first half of the 18th century and the didactic challenges and innovations introduced by Poleni.

Lecture notes and correspondence show that Poleni gave imagination and diagrams a prominent cognitive role and used geometrical instruments in teaching Euclid's *Elements* and, more interestingly, in differential calculus. In the late 1720s, he designed two instruments to trace transcendental curves: the tractrix and the logarithmic curve. These devices were first described in a 1728 letter to Jacob Hermann and published in the *Epistolarum Mathematicarum Fasciculus* (1729), accompanied by endorsements from leading Italian mathematicians. Conceived as mechanical solutions to inverse-tangent problems, the machines offered a continuous motion method for constructing curves that had resisted geometric representation. Through a wheel-guided mechanism, Poleni sought to give transcendental curves the same epistemic status as conic sections, in line with the Cartesian ideals of geometric exactness.

Although the original machines are lost, surviving syllabi suggest that Poleni may have used similar instruments to introduce calculus to beginners in his lectures. While "infinitesimal analysis" (as calculus was then called) does not appear in course lists, the syllabi indicate that he introduced elements of differential calculus, particularly in connection with physiological topics such as the "motion of animals." This unusual subject–no longer considered part of mathematics–was inspired by Jakob Bernoulli's study of muscle shapes using differential equations. It is possible that Poleni employed his mathematical machines to illustrate the concept of a "differential curve" as a solution to an inverse-tangent problem. By tracing the role of Poleni's instruments in experimental philosophy and mathematics, this paper sheds light on how 18th-century pedagogical tools bridged conceptual and disciplinary boundaries. His use of mechanical devices exemplified a visual and experimental approach to mathematics teaching and reflected broader Enlightenment efforts to unite abstract reasoning and material cultures.

Tensions dans l'élaboration des documents curriculaires dans les années 1990 : les relations entre le Brésil et la France

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Cet article vise à analyser les tensions impliquées dans la production de documents curriculaires de mathématiques dans les années 1990, au Brésil et en France, dans la définition des savoirs pour l'enseignement et pour la formation des enseignants. Il s'agit d'une recherche historique qui mobilise des études brésiliennes et françaises, valorisant le rôle de l'historien, le travail en archives et la critique des sources. Le cadre théorique articule les travaux de Hofstetter et Schneuwly sur la notion d'expert et la production de savoirs, ainsi que ceux de Bourdieu sur les champs et les agents. Le corpus considère, pour le Brésil, les *Parâmetros Curriculares Nacionais* (PCN) de Matemática pour l'enseignement fondamental, et, pour la France, le Programme de mathématiques de la classe de troisième pour le collège. Les textes curriculaires sont abordés comme des "boîtes noires" (Latour, 2000), issues de processus d'élaboration traversés par des tensions entre différents champs, et comme des monuments à interroger avant qu'ils ne deviennent des documents (Le Goff, 1990). L'analyse inclut également la législation en vigueur dans chaque pays à l'époque, des lettres, des avis, des comptes rendus de réunions et d'autres pièces des "coulisses" de l'élaboration. Dans les années 1990, le Brésil et la France ont réformé leurs systèmes éducatifs et produit de nouveaux référentiels curriculaires. Au Brésil, dans un contexte d'ouverture au marché international et d'implication du FMI et de la Banque mondiale, la Constitution de 1988 et la LDB de 1996 ont ancré la fixation de contenus minimaux et une formation de base commune ; de là émergent les PCN de Mathématiques, instrument normatif articulant une tradition de standardisation curriculaire et des agendas d'efficacité/gestion (Brasil, 1988 ; 1996 ; Lessa, 2012 ; Tavares, 2019 ; Metz & Silva, 2023). Les PCN ont été élaborés au Ministère de l'Éducation par des équipes de coordination et des experts responsables de la rédaction, appuyés par des conseillers et consultants, et soumis à une large consultation nationale avec des centaines d'avis individuels et institutionnels ; les versions ont circulé sous forme d'ébauches, de synthèses et de révisions jusqu'à la publication échelonnée. Les tensions récurrentes ont porté sur la résolution de problèmes, la place de l'histoire des mathématiques, la faisabilité et l'équité de l'usage des TIC, la progression et la portée des fractions/nombres rationnels, ainsi que sur des absences revendiquées ; s'y ajoutent la formation des enseignants, les conditions matérielles d'implantation (ressources, infrastructure) et des questions de représentativité. En France, le programme de troisième a été produit sous la coordination scientifique du Conseil national des programmes (CNP) et administrative de la DESCO, avec une rédaction assurée par le GTD de mathématiques : projet soumis à consultation nationale, retours des avis et synthèses des académies, lettres de convocation et réunions techniques avec inspecteurs, universitaires et enseignants, suivies de révisions et interventions institutionnelles. Les tensions les plus fréquentes ont concerné le rapport densité des contenus/temps scolaire, la présence des vecteurs/transformations géométriques, la clarification des compétences, ainsi que les TIC, l'équipement et la formation des enseignants. La comparaison entre les PCN brésiliens et le programme français montre que les curricula des années 1990 sont des artefacts négociés, et non de simples textes techniques : ils résultent de disputes entre l'État, les champs disciplinaires et le champ

professionnel enseignant, médiées par des experts qui systématisent les savoirs issus de ces confrontations. Malgré des trajectoires d'élaboration distinctes, les deux documents présentent des tensions convergentes : densité/temps, place des compétences, choix de contenus, TIC et formation des enseignants.

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The Articulation between History of Mathematics, Digital Technologies, and Argumentation: A Teaching Experiment Proposal Using a Diagram from Savasorda's *Book of Geometry*

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This research is part of the current investigations that we have been developing on the theme of the articulation between History of Mathematics and Digital Technologies (Bortoli, 2025). Studies carried out on this topic permeate the area of research in Mathematics Education, both in the Brazilian and international scenarios. As an example, we can consider the discussions held in the text "Uso de un diagrama histórico como fuente de aprendizaje de la geometría" (Fortuny, Gayarre, García 2025), which objective was to report how one progresses from visualization to argumentation based on a learning trajectory designed using a historical diagram as part of the geometry learning process. Thus, inspired by the aforementioned work of (Aymemí, Gayarre, García 2025), we used as a basis the *Livro de Geometria* de Abraham bar Hiyya (between 1065-1136), known in Latin as "Savasorda" (Guttmann 1903; Millàs 1931). We turn our attention to a sequence of images used to calculate the area of a circle. The activity is designed around a sequence of images (Savasorda's historical diagram), in which he attempts to demonstrate the formula for the area of a circle. The goal is to investigate how high school students (or those in the last years of elementary school) can progress from visualization to argumentation. The hypothesis of the study carried out by Fortuny, Gayarre and García (2025) is that: "there is a parallelism between the transformation in the sequence of images and the figuration processes implicit in geometry. In the progress from visualization to argumentation, we place great importance on learning mathematical processes related to the act of argumentation. Therefore, we will characterize students' learning through the progress of their argumentative actions." This teaching experiment proposal corroborates with our line of investigation. In the work "Do Clássico ao Digital: Demonstrando a Proposição 41 de Euclides com GeoGebra" (Bortoli, Batista 2025), the authors point out that the teaching experiment allowed them to explore, in a qualitative and situated way, the potential of the pedagogical use of digital technologies, in particular GeoGebra, to mediate the understanding and construction of meanings around Proposition 41 of Euclid's *Elements* by elementary school students. More than simply transposing mathematical content into a digital environment, what we observed was the emergence of a formative space in which mathematical proofs became a living, dialogic, and investigative practice. Therefore, the development and application of this teaching experiment using the historical diagram will compose a set of investigations that advocate for the use of digital technologies for mathematical proofs and demonstrations.

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The Borel exhibition at the IHP: historical and educational issues

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In 2021, at the occasion of the 150th anniversary of the birth of Émile Borel (1871–1956), the Institut Henri Poincaré in Paris organized an exhibition dedicated to its founder. It was designed by six historians of science (A. Bernard, M-C. Bustamante, M. Cléry, C. Ehrhardt, H. Gispert, L. Mazliak). Entitled "Émile Borel, a mathematician in the plural," the exhibition presents Borel's mathematical work in diverse aspect: disciplinary, but also at the interface of applications, particularly in the humanities, as well as didactic and institutional. Moreover, the panels could be used as a support for visits by secondary school students accompanied by their science teachers. This last aspect represented a challenge, insofar as it was not easy to convey the modernity of Borel's work to an audience rather distant from the issues concerned. A demanding task of popularization had to be carried out, both scientific and historical, aimed at placing Borel and his work in the social and political context of his time.

During the six months the exhibition was on display at the library, the Institute's communication department organized class visits; some were guided by us. Short practical internships for secondary school teachers were also organized, requiring the expertise of the historians, science mediators, mathematicians and trainers who organized the exhibition. Based on a series of informative texts about Borel and on the understanding of the choices of the exhibition's scientific committee, the trainees were led to develop a coherent pedagogical project in which the visit of the exhibition would be included.

All this was only the beginning of the exhibition's adventure. Following its presentation at the IHP, requests came for it to be loaned out and presented in other places, first in France and then in several other countries through translations. The exhibition currently exists in seven languages (including Portuguese, as can be seen during the present conference). The loan system offered by the IHP's communications department is simple and particularly effective. The exhibition was moreover an occasion for some talks on Borel or other figures related.

The undeniable success of the exhibition raises some questions which are worth considering more systematically:

1. What is the interest of connecting the biographical approach with the emphasis on the numerous collectives associated to Borel's career? What kind of image of mathematics does it creates for the general public?
2. To what extent does the colorful nature of Borel's activity help to catch the attention of a sufficiently wide audience in connection to the variety of their interests? For the teachers and their pupils, what does it imply in terms of pedagogical goals?
3. How can one connect the contents of the exhibition with aspects students are more familiar with? What is the role of teachers, mediators and guides in this respect? As well as the importance of the choice of place for the exhibition?

The Culture of *Sangaku* in Japan During the Edo Period

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Sangaku are like votive tablets dedicated to shrines and temples after solving mathematical problems in Japan during the Edo Period (1603-1868). No other country possesses such a culture; it is unique to Japan.

Students studying mathematics in the Edo period belonged to schools. When students solved an excellent mathematical problem, they thanked the gods or Buddha, wrote the problem and the formula for solving it on *sangaku*, and dedicated it to a shrine or a temple. In the Edo period, shrines and temples were the most crowded places, not only for festivals, celebrations, and funerals, so by displaying *sangaku* there, students could show many people the problems that could be solved.

A typical *sangaku* is constructed of wood and colourfully presented using bright pigment paints.They ranged in size, some being 20 cm in length and others over 1 metre. For example, the *sangaku* for *Zenkoji* Temple measures 320 cm in width and 133 cm in height, making it very large. The traditional Japanese text they contain is read from right to left and top to bottom. They are written in the *kanbun* language, a form of classical Chinese with Japanese readings. Tablets may be divided into several sections, each dealing with a geometrical problem. These problems challenge the reader to find some length, area, or diameter relating to the diagram in terms of other magnitudes. Problems inscribed on sangaku were considered better the fewer characters they contained.The *sangaku* are still preserved in Japan (More in the north, fewer in the south). The spread of sangaku reveals the spread of schools.

In Japan, there are currently several modern *sangaku* contest. One such event is the "Sangaku 1, 2, 3" competition, where outstanding entries are displayed to *Tōdaiji* Temple every year. Problems without definitive answers are also welcome. The current problem posed is: "The curly hair of the Great Buddha is called spirals hair, shaped like a snail shell. If one of these spirals were stretched out, how many meters long would the hair be?" It's a question born from children's unique thinking and is quite fascinating.

In the future, I would like to deepen my consideration of the spread of the schools of Japanese mathematics during the Edo period. I am also concerned that many of *sangaku* are not in a good state of preservation. For example, the characters on some *sangaku* have faded and are no longer legible. *Sangaku* and books on Japanese mathematics from the Edo period must be handed down to future generations as part of Japan's traditional culture. Then we aim to promote *sangaku* contests and preserve *sangaku* culture.

The Educational Value of the History of Mathematics for Learners with Intellectual Disabilities

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Making mathematics accessible to all (Schubring, 2014) was explored for individuals with intellectual disabilities by Édouard Séguin since the 1840s (Gil Clemente, 2025). Yet still now there is a need to rethink both the curriculum and teaching methodologies.

A longitudinal study conducted over the past decade in Spain with children and teenagers with Trisomy 21 has highlighted the potential of working with geometry to enhance their understanding and knowledge of the world (Cogolludo-Agustín & Gil Clemente, 2019). When selecting content and methods that enable to engage with mathematics under higher expectations regarding their performance, the value of mathematics for human flourishing –beyond its practical applications–is a key perspective (Faraguer & Gil Clemente, 2019). In recent years the feasibility and suitability of introducing history of mathematics is being considered. The working hypothesis is that history could help revealing the human roots of number, measure, and form– thus confronting the educational challenge regarding mathematical objects of "[leaving] the embryo to mature, [...] [leading] it to consciousness, thus to give it a meaning, [...] conferring on it existence in the mental world" (Thom 1973, p. 202).

A case-study research has been carried out in Italy to test young adults in a PFI-special needs class (15 pupils, aged 16-29), in the public system of education (Passacantilli, 2022). A learning path was designed aimed to cope with their explicit aspiration to know more about mathematics and to understand roots/origins of the surrounding world. Activities regarded the origins of counting, measure and numerical notation. An approach to history was implemented, also including mythical understanding (Egan, 2014/1988: storytelling, binary opposites, metaphors) of the past.

Over the past year, a second case-study is under development in Spain (5 learners aged 16-19) in a leisure time series of workshops along the same contents and adding ancient Egyptian arpedonaptai.

The expected and unexpected results will be discussed: an improved grasping of time, space and "the ancients"; deepening of number conceptions; pupil's shared expectations regarding understanding of mathematical ideas were met.

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The effect of solving historical Fibonacci problems on students' beliefs about the nature of mathematics

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This paper presents the results of an exploratory experimental study on the inclusion of the history of mathematics in teaching and its effects on students' beliefs about the nature of mathematics. High school students participated in the study. They were divided into four groups: two control groups (85 students) and two experimental groups (88 students). The control groups worked in a traditional way, without solving historical problems. The experimental groups were given six historical problems to solve, found in the book "Liber Abaci," written by Fibonacci. To measure the impact of this intervention on students' beliefs, a survey was designed with six statements. These statements address the nature and activity of mathematics, as an exact science connected to reality, and compare what mathematicians used to do with what students do today. For each statement, students were asked to justify their answer. The survey was administered to all four groups before and after the intervention, as a pre-test and post-test. The six historical problems were chosen with a view to how they might influence belief change. The intention was that the students, after solving the problems and being given Fibonacci's solutions, would compare their solutions with those given by this famous mathematician. The students were shown the mistakes Fibonacci made, to demonstrate that mistakes are part of learning and the evolution of a science, and that this evolution has involved a change in solution strategies and methods. Also, through the discussion, it was concluded that when solving a problem, not only mathematical knowledge is involved; reality must be considered to arrive at an answer that is not only mathematically correct, but also realistic. Comparing the survey results, changes are observed in both groups. Greater positive changes can be seen in the experimental groups. The survey results, especially the content of the arguments, show the influence that Fibonacci problem solving had on the experimental groups. The students' problem-solving processes show that historical problems, in addition to providing a glimpse into the past, can also be used to encourage students to reflect on the mistakes made by mathematicians throughout history and to propose their own paths to solutions. This allows students to develop creative thinking and connect mathematics to real life, which motivates learning and enriches teaching. These results justify the inclusion of the history of mathematics in mathematics education through historical problems.

The History of Mathematics as a Critical Tool in Pre-service Teacher Education in Brazil

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3

The relationship between history, mathematics and education has been understood in the community of mathematics educators as a way of understanding mathematics as a human endeavour, the result of contributions from different cultures, and therefore more than just a set of finished results (Chorlay, Clark & Tzanakis, 2022). In other words, the integration of historical and epistemological issues into mathematics education is a way of working with mathematics as a field in construction, which can help to understand it as a science that has undergone changes over time, both in terms of what is understood as mathematics and how it is taught. Furthermore, this construction has been underpinned by ongoing dialogue with other disciplines and has sustained scientific and social developments. In the Brazilian context, Souza (2023) points out that the history of mathematics has been taught as a course in mathematics teacher training for approximately 40 years, which provides a fertile environment for the aforementioned integration. In this context, the perspective of Critical Mathematics Education, as discussed by Skovsmose (2023), questions the ideology of certainty, which places mathematics as universal and valid regardless of historical, social, and political contexts. Thus, this work analyses a educational intervention in a History of Mathematics course in initial teacher training, with the aim of discussing how this approach can support the development of a critical stance in line with Critical Mathematics Education. As an example, we discuss one of the interventions carried out in the course, whose objective was to explore the concept of accuracy, based on the history of units of measurement and their precision. The activity addressed issues related to the standardisation of the metre throughout history and the use of instruments such as the calliper, especially with the application of Vernier. At the end of the activity, students asked other questions about the history of other measurement standards, such as the kilogram, which indicate an understanding of mathematics as a human, historical, and socially relevant activity. As a result, this suggests that the history of mathematics, in this sense, began to function as a critical tool against the ideology of certainty (Skovsmose, 2013).

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The history of Mathematics Education in Teacher-training of Secondary Education in Cabo Verde

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3

The study of recent history is important, for a comprehensive understanding of current teaching, identifying strengths and weaknesses of different historical period.

Historically, in all educational systems, the discipline of mathematics has always occupied a prominent place, thus being a good barometer for measuring the degree of development of an educational system. In this context, we present this proposal. based on research developed within the scope of my master's dissertation, whose theme aligns with the aforementioned title. The work was supervised by Professor Dr. Helmuth Malonek. from the university of Aveiro. and Dr. Hélder Pinto, from the Piaget Institute of Vila Nova de Gaia.

Through this brief presentation, we intend to analyze the evolution of mathematics education in teacher training for secondary education in Cape Verde, from the early days of national independence (July 5, 1975) to the present day. Specifically, we aim to describe in detail the structure of the mathematics courses in the early post-colonial years, as well as identify the key figures and institutions that formed the basis of this project. Furthermore, we will briefly present accounts and testimonies from teachers and former students who attended the Teacher Training School for Secondary Education.

The history of symmetry – from art, via crystallography, to mathematics

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4

While symmetry has fascinated humans from the very earliest civilizations, the mathematical study of symmetry is comparatively recent. While mathematics was formed as an exact science in ancient Greece, in particular by the famous Elements of Euclid, and even if the Elements and results of other ancient Greek mathematicians are sufficient for studying the basic geometric properties of symmetry, in spite of the word symmetry itself being of Greek origin – the first mathematical definition of (mirror) symmetry appeared as late as 1794 (A. M. Legendre, *Éléments de Géométrie*). However, during the renaissance Leonardo da Vinci studied symmetrical arrangements for art purposes, and the first classification of symmetries is usually attributed to him. While it is safe to say that early studies of symmetry arose from art, during the 17th and 18th century a new impulse came for studying and classification of symmetries – from crystallography, an emerging scientific discipline. This culminated in classifications of symmetries in two and three dimensions (most notably by German physician and mineralogist J. F. C. Hessel in 1830, and French physicist A. Bravais in 1850). At about that time mathematicians working in the new discipline of group theory, most prominently C. Jordan towards the end of the 1860es, noted the possibility of approaching symmetries using groups. This culminated towards the end of the 19th century both in group theory becoming a fully developed mathematical discipline in its own right, and the famous enumeration of all 230 possible symmetrical arrangements in crystal structures by A. M. Schoenflies and Y. S. Fyodorov (1891). While mathematical interest in symmetry turned towards higher dimensions during the first decades of the 20th century, the crystallographic interest seemed to decline – until the recent discovery of quasicrystals.

In this talk we shall describe the main moments of the fascinating history of the mathematics of symmetries, with emphasis of moments and ideas relevant for teaching about symmetries to university students of the sciences (chemistry, physics, geology), university students of mathematics, but to some part also to students of secondary level general education in the mathematics, or chemistry, classroom.

A large part of the presentation will concern joint work with Vladimir Stilinoviȁ (Univ. of Zagreb).

The Representation of the History of Mathematics in Turkish High School Textbooks: A Comparison of the 2019-2024 Curriculum and the 2025 Türkiye Century Maarif (Education) Model

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2

This study investigates the representation of the history of mathematics in Turkish secondary school mathematics textbooks, focusing on a comparison between the 2019-2024 curriculum and the 2025 Türkiye YüzyılıMaarif Model. While mathematics education has often been limited to technical skills and problem solving, the integration of history of mathematics provides students with a broader cultural and epistemological perspective. Previous research has emphasized that history of mathematics can enhance students' motivation, support conceptual understanding, and strengthen interdisciplinary connections.

The study employed a qualitative document analysis methodology. Six textbooks were examined: three published under the 2019-2024 curriculum and three prepared within the framework of the 2025 Türkiye YüzyılıMaarif Model. The historical contents in these textbooks were categorized into three dimensions: unit introductions, topic explanations, and end-of-unit sections. The analysis revealed a total of 91 historical elements. Of these, 53 were identified in the 2019-2024 textbooks, while only 38 appeared in the 2025 textbooks. Furthermore, a notable shift was observed in the placement of these contents: during 2019-2024, history of mathematics was mainly embedded within topic explanations, whereas in 2025 textbooks it was predominantly positioned at the end of units.

These findings suggest that while the new curriculum ensures standardization, given that all textbooks are produced centrally by the Ministry of National Education, it also limits the diversity and richness of historical content. This raises concerns about the practical use of history of mathematics in classrooms. When historical notes are placed outside the core of the lesson, there is a higher risk that teachers may neglect them during instruction.

This study contributes to international discussions on the integration of history of mathematics in classrooms by providing evidence from a large-scale curriculum reform. It underlines the importance of systematic and continuous inclusion of historical perspectives in textbooks and highlights the need for teacher training and support materials to ensure the effective pedagogical use of history of mathematics. The findings aim to inform both curriculum developers and researchers working at the intersection of mathematics education and history of mathematics.

The role of AGDs in proving old problems

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Euclides' work is a remarkable legacy in mathematics history.

Over the centuries, several mathematicians have continued this work, demonstrating important results, such as the geometry of the triangle, including its notable points. Euler's proof that the three notable centres of the triangle are always collinear regardless of the triangle chosen, being known as Euler's line.

Proof is a powerful mathematical activity that requires validation, explanation, discovery, systematisation of results and transmission of mathematical knowledge. The process of proof allows new results to be discovered and mathematical knowledge to be communicated. Geometric proof is widely recognised as a central component of mathematical education, as acknowledged by Ball et al. (2002), Mariotti (2006), and Stylianides & Stylianides (2017) because it incorporates tools, methods, and strategies for problem solving. However, integrating proof into the teaching and learning process of mathematics remains a challenge (Stylianides & Stylianides, 2017).

The current essential learning (AE) for Mathematics A in secondary education revisits some of these theorems, recommending their demonstration. The role of proof and the act of proving in mathematics leads us to the need to teach proof in mathematics lessons.

With the aim of encouraging and challenging future mathematics teachers to integrate the proof of Euclidean geometry problems, multiple proof tasks (MPT) (Leikin, 2009) were proposed, requiring different representations. In the proof process, future teachers had access to GeoGebra to construct the figures involved. The visualisation and manipulation of the constructed figures have supported the Euclidean proof.

Arzarello et al (2009) question the extent to which the notion of proof becomes different in a Dynamic Geometry Environment (DGE). This is the central issue in this paper. It is debated whether visual representations should be treated as complements to proofs, as an integral part of proofs, or as proofs in themselves. It was considered that they are indispensable in the process of learning proof. On the other hand, the emergence of Dynamic Geometry Environments (DGE) has transformed traditional Euclidean proofs into dynamic, manipulable proofs.

Several authors warn that the completeness of formal proofs is closely linked to their degree of complexity, which can make them difficult for students to understand. Thus, different levels of proofs can lead to different types of situations for which different forms of proof are appropriate (Dreyfus et al, 2012). In this paper, we show how different types of proofs, dynamic or Euclidean, take on different didactic functions, facilitate understanding, and make the proof more accessible, allowing students to become more involved in the proof activity and gain a better understanding of the proof and the learning process involved.

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The Rule of Three in *Recknekonsten* – Hans Larsson Rizanesander's treatment of problems of proportion

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This paper examines the rule of three as presented by Hans Larsson Rizanesander (1574-1646) in his 1601 manuscript *Recknekonsten* [The Art of Arithmetic]. The handwritten work represents an important cultural artefact in the early history of Swedish mathematics education. It is the oldest known manuscript on arithmetic written in Swedish, composed at a time when formal education was reserved for privileged boys educated in Latin. Although Recknekonsten was never printed in Rizanesander's lifetime and its actual use remains uncertain, it provides valuable insights into the early development of mathematics teaching in Sweden.

The *Art of Arithmetic* presents Hindu-Arabic notation and abacus practice, followed by, among other topics, the fundamental operations for whole numbers and fractions, and treatments of numerical series and root extraction. Of particular interest in this paper is Rizanesander's treatment of the rule of three. This section contains numerous worked examples but no explicit explanations of the underlying principles.

Historically, the rule of three was used to solve problems based on proportions. Given three quantities a, b, and c such that $\frac{a}{b} = \frac{c}{x}$, (i.e., $a : b :: c : x$, the unknown quantity x can be found using the formula $x = \frac{c \times b}{a}$. Rizanesander expresses the rule verbally:

"The first and the last or the third should have the same name, and likewise the middle or the second and that one wants to know. [...] Multiply the last with the middle, divide the result by the first: the quotient is the answer."

An example provided by Rizanesander reads as follows:

"One buys $\frac{3}{4}$ cubits of fabric for 12 öre. How much is 9 cubits? Answer 18 mark. Put it in the rule:

$\frac{3}{4}$ cubits - 12 öre - 9 cubits.

Multiply the last 9 with the middle 12 makes 108 and the same 108 shall be divided by the first which is $\frac{3}{4}$ [...] The quotient is 144 öre and the same 144 öre make with 8 to mark, makes 18 mark and so many mark is the cost of 9 cubits."

Rizanesander's presentation of the rule of three is not based on proportional reasoning but on algorithmic and mechanical computation, demonstrated through numerous examples. The problems reflect a broad spectrum of early modern commercial and practical contexts. They concern the prices of materials, such as fabric and silk string, as well as commodities such as wax, raisins, ginger, butter, and beer, and metals including copper, iron, and lead. Several examples also deal with time and measurement, such as wages, the length of shadows, or the depth of a well. Throughout these examples, Rizanesander employs local Swedish units of money, weight, and length, grounding the computations in the material culture of everyday trade. His problems are expressed through both whole numbers and fractions, showing a consistent attention to numerical detail and calculation. In this paper, we analyse several of these examples to highlight the computational procedures and discuss influences from earlier or contemporary authors such as Christopher Clavius (1537-1612) and Petrus Ramus (1515-1572).

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The use of rhetoric and syncopated algebra in pre-service teacher education

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Elementary algebra belongs among the most difficult topics in mathematics education at the primary and secondary level. Research indicates that pupils do not like algebra, and school algebra produces in them anxiety. I believe that one of the reasons is that the teaching of algebra in primary schools starts with the introduction of letter notation that is equivalent to the one introduced by Descartes in 1637. Thus 700 years of history, the era of rhetoric and syncopated algebra, which starts at least with al-Khwarizmi, is ignored by the curricula.

Even if there is little chance to change the curricula and introduce some hours dedicated to rhetoric algebra, it seems reasonable at least to make teachers aware of the problem, so that they can make in their teaching some corrections. Therefore I have designed a one semester course on history of algebra for pre-service teacher education. In the course we are reading sources of rhetoric and syncopated algebra and then solving problems formulated in that style, using only tools found in the sources. (I am aware that among historians there are doubts concerning the notion of syncopated algebra (e.g. Albrecht Heeffer), but this notion is explanatory, as for instance in physics this style of notation is used until our days. $F = m \times a$ means *f*orce equals *m*ass times *a*cceleration).

In the course I concentrate on the main epistemological shifts that occurred in the algebraic notation and trace them until Descartes, i.e. until the standard notation taught to the pupils. In each case I indicate the text used to discuss the innovation.

In use of letters these are: 1. Introducing letters to denote unknown quantities (Pacioli 1494); 2. Subordinating letters denoting unknown quantities to arithmetic operations (Pacioli 1494); 3. Expressing identity of the unknown when writing its powers (Viète 1591); 4. Inventing a way of introducing second and further unknowns (Viète 1591); 5. Overcoming homogeneity, which prevents the addition of different powers of the unknown (Descartes 1637).

In operations with letters these are: 1. Introduction of notation for steps of an infinitely repeatable operation of exponentiation (Pacioli 1494); 2. Replacement of these verbal names with letters (Widmann 1489); 3. Introduction of the power indicator as a right superscript (Regiomontanus 1463); 4. Identification of the sequence of symbols for individual powers with the number series (Chuquet 1484); 5. Introduction of convention expressing the identity of the unknown (Viète 1591); 6. Combination of the expression of the identity of the unknown with an index denoting its power (Descartes 1637).

In a similar vein we discuss the road leading to the conventions used in the designation of the operation of root extraction (leading from Regiomontanus to Descartes); the way of writing equations (the different styles of what to put on the right hand side and what on the left (connecting Pacioli, Stifel, Viète and Descartes); the problem of negative values for coefficients as well as solutions.

In this way I hope to arise in the students an awareness of the rather complex issues hidden in the seemingly so simple and transparent Cartesian algebraic notation. In my presentation at the Summer University I would like to present the epistemic shifts that occurred in algebraic notation and discuss them with the audience. I am curious whether I have forgotten some important change, as well as whether there are perhaps different roads leading to the same shift, or better texts to illustrate them.

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The "GIRP", An international sect of New Math enthusiasts in the post-New Math era?

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In 1971, Georges Papy and Frédérique Papy-Lenger, the leading couple in the Belgian New Math (or modern mathematics) movement, founded the "Groupe International de Recherche en Pédagogie de la Mathématique" (GIRP)/International Group for the Study of the Pedagogy of Mathematics (De Bock & Vanpaemel, 2019). Actually, the GIRP was a split off from the Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques (CIEAEM)/International Commission for the Study and Improvement of Mathematics Teaching, which Papy had left in 1970 because a majority within the CIEAEM was no longer willing to support his New Math views. Several unconditional New Math believers of the early 1970s followed the Papys.

As a rationale for founding the GIRP, the initiators stated that the intention was to bring together people who were fully focused on research in the field of (modern) mathematics education. The objectives of the GIRP were specified in its Articles: "The association aims to promote human and professional exchanges between mathematicians, psychologists, educators, and users of mathematics from all countries, and to organize international meetings, allowing researchers in mathematics education to exchange the results of their work" (Groupe International de Recherche en Pédagogie de la Mathématique, 1971, p. 7). At the first meeting of the GIRP in Luxembourg (July 19-28, 1971), an executive committee was elected with Frédérique Lenger (Belgium) as president, Robert Dieschbourg (Luxembourg) as secretary, and Chris De Munter (Belgium) as treasurer. Lenger will serve as president of the GIRP until 1981.

The GIRP will organize a total of 28 meetings in different places in Europe between 1971 and 1999. In this presentation we describe and analyze the actions of the GIRP. The first four meetings (1971-74) are properly documented in the journal Nico. Main points of interest during that period were (1) the reform of mathematics education at the primary level, (2) the use of the results of scientific research in the pedagogy of mathematics by in-service teachers in non-experimental classes, (3) the mathematical-logical foundation of modern mathematics education, and (4) modern mathematics as a therapeutic and educational tool for supporting mentally or physically disabled children. Hardly any documents exist for the period 1975-92. From 1993, under the presidency of Bruno D'Amore (and from 1996 of Carla Caredda), reports of the GIRP meetings are available in the *Rivista Scuola ticinese* (1993-94) and in the *Bollettino dei docenti di matematica* (1995-99). During this period, the focus was no longer exclusively on New Math pedagogy and even some internal critique on that pedagogy emerged (D'Amore, 2016). After meeting in Bardonecchia (Italy), in 1999, the group died a quiet death.

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Ubiratan d'Ambrosio at the Brazilian colloquium of mathematics (1957): interlocutions, knowledge, and the constitution of mathematics education in Brazil

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This article analyzes the early teaching experiences of Ubiratan D'Ambrosio (1932-2021) in Brazilian higher education, emphasizing his participation in the First Brazilian Colloquium of Mathematics, held in 1957, and its significance for the constitution of Mathematics Education as a scientific and academic field in Brazil. The investigation, based on the crossing of historical sources, correspondence, testimonies, and records produced between the 1950s and 1960s, seeks to understand how networks of interlocution, academic practices and processes of knowledge circulation contributed to teacher education and to the emergence of an epistemology oriented toward educational practice and the cultural dissemination of mathematics.

The theoretical framework is grounded in the perspective of Cultural History, particularly the contributions of Roger Chartier and Michel de Certeau. Chartier supports the understanding of processes of appropriation and representation of scientific knowledge, while Certeau provides the category of "strategy" to interpret teaching practices. From this theoretical dialogue, D'Ambrosio's teaching is understood as a space of creation, in which mathematics education is configured as a cultural practice socially situated and historically conditioned. D'Ambrosio's early teaching experiences took place in a context of reorganization of Brazilian mathematics, characterized by the need for institutional strengthening, knowledge circulation, and the construction of a scientific community.

Within this scenario, the First Brazilian Colloquium of Mathematics, held in Poços de Caldas (MG) and coordinated by Chaim Samuel Hönig, constituted a privileged space for scientific and teacher education. The event brought together 49 participants from different university centers, such as USP, ITA, and IMPA, providing young professors, including D'Ambrosio, with contact with themes from different areas of knowledge and with researchers who would play an important role in the consolidation of mathematics in the country. The interlocutions established during the Colloquium, later deepened through correspondence with Brazilian and foreign mathematicians, reveal a formative process that goes beyond the technical dimension of mathematics.

The analyzed letters and testimonies highlight criticisms of transmissive teaching, curricular fragmentation, and the absence of a conceptual and meaningful approach. These dialogues emphasize the importance of teacher education and of understanding mathematical knowledge. Read in light of Chartier's perspective, the interactions between D'Ambrosio and researchers such as Chaim Hönig, Elza Furtado Gomide, Lindolpho de Carvalho Dias, and Alberto Azevedo reveal processes of circulation and appropriation of knowledge that contributed to the constitution of a scientific capital grounded in cooperation and knowledge production. These exchanges also reveal the author's continuous effort to integrate Brazilian scientific production into the international scenario, reinforcing the cultural and relational dimensions of his formation. It is concluded that the 1957 Colloquium represented a milestone in

D'Ambrosio's trajectory and in the history of Brazilian Mathematics Education. His experiences at this event and in the networks that emerged from it enabled the development of a critical and inventive teaching practice, committed to understanding mathematics as a cultural practice.

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Using Historical Sources to promote reflections on Rigor and mathematical Reasoning in High School: Area Determination as Case

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In the spring of 2024, an experiment was carried out in a Danish high school class to test if students' inquiry of the past can be used to promote their reflections on the present with respect to rigor and reasoning in mathematics. Determination of areas was used as case. The students worked with Archimedes' "The Method" and Newton's "Rule 1" for calculating areas under simple curves during the teaching experiment.

In the talk, we introduce the "methodology that was created and used in the experiment (Milthers & Wedderkopp 2024). The activity was developed based on the Anthropological Theory of Didactics and Study and Research Paths (SRP) (Chevallard & Bosch 2020), merged with the historiographical frameworks of epistemic objects, techniques, and configurations of mathematical "(Rheinberger 1997, Epple 1999) and a multiple perspective approach to history of mathematics (Kjeldsen 2023). The design was inspired by works on how to use historical sources to create an inquiry-reflective learning environment (Johansen & Kjeldsen 2018), which links to the idea of designing the milieu with specific media or resources for SRPs (Jessen 2017).

The design and implementation of the SRPs in the classroom will be presented and discussed together with the findings which indicate how to promote students' reflections on rigor and mathematical reasoning through their engagement with the historical sources and exhibit limitations in the students' understanding of rigor. We see rich perspectives for further research on using the past to orient oneself in the present with respect to developing students' reflections on rigor and their competences in mathematical reasoning within the didactical and historiographical frameworks used in the present study.

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Vector Calculus in Brazil and France (1900-1930): A historical-mathematical perspective based on pioneering works

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Entering the field of Mathematics Education, the present work positions itself as an integral part of the history and historiography of mathematics in Brazil and its relations with other countries, particularly France, within the time frame extending from 1900 to 1930, with regard to vector calculus. Likewise, it is also aligned with research aimed at broadening the discursive presence of Brazilian figures and works that played a significant role in the history of science and mathematics in our country and/or abroad. Based on previous studies (Bonfim & Nobre, 2021), it can be stated that vector calculus, as a discipline, was introduced in Brazil for the first time in higher education courses in 1926, at the Polytechnic School of São Paulo, under the responsibility of Professor Theodoro Augusto Ramos (1895-1935). The first work on this subject, entitled *Cálculo vectorial*, is also authored by him and was published the following year, 1927, compiling lecture notes from the previous year. This fact made it possible to situate the publication of Ramos (1927) as the first work published in the country on the subject, with the author also being the first Brazilian to publish a work on this topic in Paris, France, in 1930 (*Leçons sur le calcul vectoriel*). As for France, preliminary studies (Bonfim, 2025) highlight the pioneering nature of the work *Calcul Vectoriel. Théorie, applications géométriques et cinématiques destiné aux élèves des classes de mathématiques spéciales et aux étudiants en sciences mathématiques et physiques*, published in 1923 by Albert Châtelet (1883-1960) and Joseph Kampé de Fériet (1893-1982), with regard to vector calculus in France. This work was contemporaneous with that of Ramos and is of particular interest for this research.

Accordingly, this study seeks to present the development of a narrative grounded in the history of science and mathematics, examining the similarities and differences evidenced in the aforementioned works. Furthermore, it aims to provide an understanding of the relevance of these productions for their respective periods and national contexts, as well as to identify the main references mobilized by the authors and the possible cross-influences between their works. By adopting this perspective, the intention is to contribute to the expansion of the debate on the circulation of knowledge and the intellectual networks that shaped the development of vector calculus in the early twentieth century in the countries considered.

Consequently, this study not only positions itself within the field of historical research in Mathematics Education but also reinforces transnational dialogues, shedding light on Brazil's significant presence in this context and emphasizing its active participation in the process of constructing and consolidating scientific knowledge.

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"Who's Who in Mathematics?" Integrating the History of Mathematics into the Classroom through Gamification

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2

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The integration of the History of Mathematics into secondary education is essential for students to recognize mathematics as a human intellectual enterprise. This proposal presents the implementation of an educational game titled "Who's Who in Mathematics?", conducted in two 12th-grade secondary school classrooms (Mathematics A). The activity aims to help students recognize relevant mathematicians and their contributions to the curriculum while relating mathematical concepts to their historical and cultural contexts. The game is designed for groups of 3 to 4 students. The physical setup in the mathematics classroom involves 20 portraits or illustrations of mathematicians placed on a wire along the wall using suction cups and hooks. Each group receives an envelope containing 10 information cards that describe mathematical contributions, curiosities, or historical impacts. The students' task is to discuss and correctly associate each information card with the corresponding mathematician's portrait using clips and color-coded stickers for group identification.

The classroom dynamic lasts approximately 60 minutes and is divided into five stages:

- Introduction (5 min): Contextualization of the game.
- Preparation (5 min): Explanation of rules and material distribution.
- Development (20 min): Group discussion and association of cards.
- Scoring (20 min): Teacher-led verification and collective discussion of mathematical contributions.
- Synthesis (10 min): Identification of winners and student self-evaluation.

The evaluation of this activity is three-fold: formative, through the observation of teamwork and critical thinking; diagnostic, by identifying which mathematical contents were consolidated; and reflective, through student self-evaluation. By implementing this game in the mathematics classroom, we promote a collaborative learning environment where students value the role of history in the development of discipline.

In conclusion, this initiative illustrates the potential of gamified historical integration to foster a collaborative environment where mathematical knowledge is constructed through its historical context. Future iterations of this project aim to involve students directly in the research and creation of new content, while exploring digital formats to further integrate mathematical heritage into the modern technological landscape of mathematics education.

"6 Milestones", an Erasmus+ project to promote historical recreational problems in mathematics education

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The K2 Erasmus+ project, "6 Milestones", focuses on integrating the history of mathematics into secondary education by seeking to promote mathematics and its historical context among secondary school teachers and students. It runs from Winter 2025 to 2028, bringing together six European partners (France, Portugal, Italy, Austria, Cyprus, and Greece). It uses non-formal and recreational methodologies incorporating various digital tools to enrich the learning experience.

Along the three years of the project, we will explore six seminal works in the history of mathematics. These are the Rhind Papyrus, the Greek Anthology, Diophantus' Arithmetica, Alcuin's *Propositiones ad Acuendos Juvenes*, Fibonacci's *Liber Abbaci* and Luca Pacioli's *De Viribus Quantitatis*. The immaterial value of the mathematical principles and their sociocultural context form the basis for the project's various deliverables: the creation of an interactive e-book, the development of practical and digital exhibitions, and teacher training. The resources are being designed to be accessible to a variety of audiences, including students with special educational needs. This project's innovative approach combines mathematics, project-based learning, gamification, culture, and history in teaching sequences that are integrated into the students' traditional curriculum. This presentation will outline the project's objectives and expected results.

"Mathematics for the Million" according to Bento de Jesus Caraça

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The Portuguese mathematician Bento de Jesus Caraça (1901-1948) wrote a book that had and still has a huge impact on its readers. It is called *Conceitos Fundamentais da Matemática* (Fundamental Concepts of Mathematics) and wants to present the evolution of Mathematics as it happened really, from the early numbers to Sequences and Series.

In the introduction he wrote: "Science, viewed in this way, appears to us as a living organism, imbued with the human condition, with its strengths and weaknesses, and subordinate to the great needs of man in his struggle for understanding and liberation; it appears to us, in short, as a great chapter in human and social life."

Clearly this is a very different kind of book, partly historical, partly philosophical, partly technical.

In some parts he uses a dialogue form, in some other parts he quotes historical figures, like the well known quote of Herodotus about the birth of Geometry or a quote from Dedekind's book on Continuity and Irrational Numbers.

He tries to show how and why mathematics evolved including the use of marxist points of view like in this assertion: "Where there is evolution to a higher state, the negation of a negation is realized."

It is clear Bento de Jesus Caraça was aiming at an audience as big as possible, as he also created the so called "Cosmos Library" (that published more than 100 volumes) and that he presented as aiming "to provide as much general knowledge as possible to as many people as possible, to make accessible to everyone what the material conditions of life and the professional needs of specialization always make difficult, and sometimes even impossible, to acquire–a general vision of the world, the physical world and the social world, of its construction, of its life, and of its problems."

We will also mention the Popular University, that he helped to create in 1919 and organize, where workers would attend courses. He would justify the creation of this university as "culture must be promoted for all, and this is possible because it is not inaccessible to the masses; the human being is indefinitely perfectible, and culture is precisely the indispensable condition for this progressive and constant improvement."

Was this a similar initiative as Lancelot Hogben's "Mathematics for the Million " (1895-1975)?

Which are the similarities and the differences?

What was/is their impact today? Which other books fulfill today this goal?

Short Oral Presentation

Short oral presentations in the program are arranged alphabetically by title.

Analysis and Synthesis in teaching Calculus: Historical insights

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One of the main criticisms Jo Boaler raises about the teaching of mathematics at different levels is the lack of investigative and creative processes during learning. The development of so-called mathematical mindsets, which consist of a psychological system of beliefs that promote overcoming challenges in learning and foster positive responses to errors, requires a stronger connection to the processes of doing and creating mathematics. In this context, we propose using analysis and synthesis methods to discuss concepts from single-variable differential and integral calculus, as well as some paradoxes. Throughout history, mathematicians have used the terms "analysis" and "synthesis" to differentiate distinct styles of argumentation and exposition. There are numerous interpretations of the notions of analysis and synthesis. In the logical interpretation, analysis begins with the general and moves toward the particular, while synthesis proceeds in the opposite direction, starting from the particular and moving toward the general. In the methodological interpretation, analysis remains confined to the general level, while synthesis focuses on the particular, relating it to the general in the context of the specific or even the individual. In the configurational interpretation, in mathematical reasoning or proofs, synthesis determines the consequences of certain premises, producing a tree of successive, related deductions. Conversely, analysis identifies the functional relationships within a specified domain of entities, whether known or unknown, by transforming them into a functional configuration. In the linguistic interpretation, a mathematical argument or the formulation of a mathematical problem or proof is considered synthetic if it employs the language of classical geometry and the theory of proportions. On the other hand, it is considered analytic when it uses the language of equations, functions, or operations. In this work, we will delve into the historical use of the methods of analysis and synthesis in the context of determining tangents, maxima, and minima, among other topics. There is a perception that ancient mathematicians "hid" their methods of analysis, presenting only the results through a rigorously synthetic approach. This sense of concealment seems to persist in calculus classrooms today. We aim, for example, to explore Fermat's method for determining maxima and minima. With Leibniz, we intend to investigate the inverse tangent problem in its symbolic and geometrically constructive versions. With Barrow, we will focus on the geometric proof of the Fundamental Theorem of Calculus. We also plan to examine the paradoxes of the rectangle and Gabriel's horn by Torricelli, the paradox of Aristotle's wheel, and Fermat-Torricelli's problem, among others. Contact with these historical traditions offers the possibility of gaining a deeper understanding of the processes of mathematical production, as well as their technical and cultural nuances.

Dante, Mathematics and Pœtry in the *Divine Comedy* : Games & Maths – An interdisciplinarity approach

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The project "Dante; Mathematics and Pœtry in the Divine Comedy", aims to explore The relationship between mathematics and literature, which are generally considered antithetical and incompatible subjects which is why many students choose courses of study where there is little mathematics, believing themselves to be more suited to literary subjects than scientific one.

The Divine Comedy itself is basis for learning as well as for promoting cross-curricular approaches. It can serve as social purpose as it opens minds towards different cultures and it wil be used in the classroom to:

- Organize theatrical activities
- Encourage students to appreciate how mathematics problems are used by Dante as methaphor to express theological and philosophical conepts
- Alleviate maths anxiety by understanding history
- Encourage cross-curricular approach to the mathematics
- Connect the study of mathematics to human emotions and motivation

The activities of the project are:

Reading Paradiso Canto XXVIII, which refers to the legend of Sessa, who as reward for the invention of the game of chess asked for a grain of wheat in the first square, doubling it subsequently up to sixty-four," and I saw more (the engels) than the chessbord doubling multiplies into thousands" from this sentence the students are going to develop the concept of geometry sequence ,calculating the enormous number of wheat and the demostration of the formula of the quantity. We will talk about the game of chess in the Middle Ages, visiting the frescoes of the Davanzati Palace in Florence, where the game of chess between a lady and her knigth is represented and we will listen to some excerpts from the otiginal Carmina Burana, referring to diceplayers in medieval taverns. The other canto that will be read is Purgatorio Canto VI, which features the medieval game of Zara. This will allow us to study the basis of probability calculation and games of chance in the Middle Ages with references to the low probability of winning in contemporary games of chance; some games will be simulated with computers tools. We will study the story of Rinaldeschi, who lost at the Zara game, visiting the pictorial panels at the Stibbert museum in Florence.

Dante was not only a poet, bu also a profound conoisseur of the arts of Quadrivium: Arithmetics, Astronomy, Geometry and Music, whose knowledge is used by Dante in many Cantos of the Divine Comedy; therefore the project is going to be expanded in the following years with references to *Geometry, logic, Arithmetic and Astronomy*.

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Euclidean principles of area equivalence for geometric problems to work with undergraduate and high school students

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In this work, I discuss my experience of teaching the principles of area theory from Euclid's *Elements* to students at the State University of Rio de Janeiro (UERJ) for four semesters, in order to solve problems from Mathematics Olympiads and preparatory lists of exercises for Olympiads. Several of these problems are of the following type: finding the area of a region of a triangle or parallelogram generated by dividing the figure by line segments. Based on this experience, I began a project on the subject with a student in the PROFMAT master's (professional master's degree in basic mathematics teaching).

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Historical sources as mediators for conscious teaching Styles - A meta didactic analysis of Fibonacci's "Broken Numbers"

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This research investigates the impact of integrating the history of mathematics into teacher professional development, specifically observing the evolution of teachers' praxeologies and beliefs. The core of the study is a training program centered on the concept of fractions, explored through the historical lens of "broken numbers" (Moyon & Spiesser, 2015) as presented in Leonardo Pisano's *Liber Abbaci* (Boncompagni, 1857; D'Alessandro & Giusti, 2020). The integration of history is not treated as a mere chronological addition but as a structural element capable of reshaping the pedagogical approach to rational numbers.

The methodology follows a qualitative approach, focusing on an emblematic case study of a teacher [Morales, et al., 2017] who had never previously utilized historical sources in her practice. At the outset, the teacher expressed a common belief: that the history of mathematics is merely a motivational or introductory tool, useful for capturing student interest but peripheral to the actual learning process (Barbin, 2022). The training intervention sought to challenge this view through the integration of multiple artifacts. The process involved the critical analysis of excerpts from Fibonacci's original source, followed by systematic, reflective dialogues between the teacher and the researcher. These confrontations focused on both epistemological issues (the nature of "broken numbers") and methodological strategies, such as the selection of manipulative artifacts to be paired with the text and the optimal sequencing of laboratory activities for the students (Cerasaro & Salvatori, 2025).

The analysis of this professional evolution is conducted using two complementary frameworks: the Meta Didactic Transposition (MDT) (Pocalana & Robutti, 2024) and the Documentational Approach to Didactics (DAD) (Gueudet & Trouche, 2009). Through these lenses, we observed a significant shift in the teacher's praxeologies. The historical source acted as a powerful mediator, facilitating a transition toward a conscious teaching style where epistemological reflection guides didactic action. A key finding is the teacher's late-stage realization of the educational and ethical value of history; by the end of the experiment, she recognized history no longer as an accessory, but as a fundamental means to humanize mathematics and provide students with a deeper sense of cultural belonging.

Furthermore, this study contributes a novel theoretical result to the MDT framework: the recording of unexpected changes in the researcher/trainer's own convictions (Pocalana & Cerasaro, 2025). This highlights that the meta-didactic encounter is a truly reciprocal process that influences the professional identity of both participants. These findings underscore the necessity of collaborative research-training models to foster an approach to mathematics education that is ethically grounded and based on solid epistemological foundations.

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Imaginary Quantities and Philosophical Legitimacy: Apolinar Fola's Contribution to 19th-Century Spanish Science

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This paper marks the beginning of a broader research project focused on the two volumes of *Investigaciones filosófico-matemáticas sobre las cantidades imaginarias* (1881, 1891), authored by Apolinar Fola Igurbide, a distinctive figure in the Spanish scientific landscape of the late 19th century. These works constitute a systematic effort to legitimize imaginary quantities from both mathematical and philosophical standpoints, at a time when their acceptance and conceptual status remained contentious within the scientific community.

The primary aim is to explore how Fola, through a philosophical-mathematical lens, addresses the issue of imaginary and complex numbers, portraying them not merely as practical algebraic instruments but as conceptual entities with theoretical existence. His approach thus lies at the intersection of mathematics and philosophy, offering an ontological and epistemological interpretation of imaginaries. Fola's work seeks to transcend the traditional divide between algebraic operability and rational justification, integrating reflection, formalization, and philosophical discourse into a coherent and original framework within the context of 19th-century Spain (Fola, 1881; Fola, 1891).

This initial study is grounded in a direct textual analysis of Fola's writings, with particular attention to their conceptual architecture, discursive style, and argumentative structure. The research methodology draws on conceptual and content analysis (Rico & Fernández-Cano, 2013), applied to historical sources in mathematics education (Madrid, León-Mantero, Maz-Machado & López-Esteban, 2021). This approach enables an examination of Fola's texts through three complementary dimensions:

- Conceptual structure, focusing on the coherence and articulation of mathematical and philosophical meanings.
- Systems of representation encompassing the algebraic, geometric, and linguistic forms employed to justify imaginary quantities.
- Phenomenology, concerning the cognitive and epistemological experience underlying the comprehension of these abstract entities.

Preliminary findings suggest that Fola's reflections anticipate later epistemological debates regarding the nature of mathematical objects and the foundations of scientific knowledge. His treatment of imaginary quantities combines abstract reasoning with formal rigor, proposing an ontological legitimization that gœs beyond applied mathematics.

As a result, Fola's writings hold considerable didactic and historical significance, serving both as a source for the history of mathematics and as a resource for teaching mathematical epistemology and philosophy. His imaginarismo represents a noteworthy contribution to the intellectual history of 19th-century Spain, exemplifying the productive dialogue between philosophical speculation and mathematical formalization.

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Lazare Carnot: Mathematics Teacher's Specialized Knowledge attached to History and Epistemology

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This text is characterized as an excerpt from the historical development of geometric concepts, based on the studies and research carried out by Lazare Carnot (1753-1823), an important figure in French engineering at the beginning of the 19th century. Thus, based on Carnot's four original manuscripts (1800), which allow for the relationship between trigonometry and geometry, focusing on the historical and epistemological character, mainly with ideas about constructions and their use in the context of military engineering as a restructuring of revolutionary armies (Gillispie, Schofield, 1972). That is, through the objective of showing the historiographical configuration of the concepts mobilized in the works of Lazare Carnot between 1800 and 1806. Therefore, scientific support is sought in documentary research (Kripka, Scheller, Bonotto, 2015), linked to aspects related to historiography (Barros, 2022), with a view to elucidating the idealized path in the writings for the presentation of geometric epistemology and the construction of tasks for training, based on the Mathematics Teacher's Specialized Knowledge (MTSK), (Caldatto, Ribeiro, 2020). In particular, Carnot (1800, 1801, 1803, 1806) presented geometric constructions based on primitive figures, directing visualization from synthesis and analysis, with construction and position tables, with the epistemological prerogative of this geometry (Gonçalves, Mendes, 2025). Thus, it can be considered that the interpretations made in these productions refer to the understanding of the conceptual meanings of the geometry used by Carnot, in order to support how his studies influenced the historical and epistemological character of constructions using the correlations of these geometric figures.

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Letters, Networks, and Institutionalization: Ubiratan D'Ambrosio's Correspondence and the Creation of SBHMats

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This paper presents an ongoing historical-documentary research that investigates the role of Ubiratan D'Ambrosio in the creation of the Brazilian Society for the History of Mathematics (Sociedade Brasileira de História da Matemática ? SBHMat), founded in the late 1990s. Ubiratan D'Ambrosio was an internationally recognized scholar whose intellectual trajectory was characterized by an articulated engagement with mathematics teaching and Mathematics Education, with sustained attention to the historical, cultural, and formative dimensions of the production of mathematical knowledge. His research interests ranged from reflections on teaching practices and teacher education to historical studies of mathematics, fostering continuous dialogue with researchers in these fields both in Brazil and abroad. The correspondence he produced and received reveals systematic exchanges with scholars in the History of Mathematics and Mathematics Education, addressing theoretical approaches, institutional articulations, and the need to establish a scientific society specifically dedicated to the study of the History of Mathematics, which ultimately led to the creation of the Brazilian Society for the History of Mathematics. This study is grounded in the assumption that personal archives–especially epistolary exchanges–constitute privileged sources for understanding the processes of institutionalization of scientific fields, as they provide access to academic articulations, institutional negotiations, and networks of sociability that often remain invisible in official records. In this regard, the research is based on the analysis of correspondence preserved in the Personal Archive of Ubiratan D'Ambrosio, currently housed at the Center for Scientific and Pedagogical Memory of Mathematics Teaching (CEMAT). The guiding research question is formulated as follows: in what ways did the academic networks of sociability established by Ubiratan D'Ambrosio, as identified through his correspondence, contribute to the articulation of ideas, individuals, and institutions that enabled the creation of SBHMat? Based on this question, the central objective is to understand the academic, institutional, and scientific processes and dynamics that supported the consolidation of the History of Mathematics as an autonomous field in Brazil, taking SBHMat as a key institutional landmark. From a theoretical-methodological perspective, the study adopts the framework of the social production of knowledge, as proposed by Peter Burke (2016), considering the stages of knowledge collection, analysis, dissemination, and use as interconnected dimensions of scientific practice. The research also engages with the concepts of scientific field (Pierre Bourdieu, 2004), institutionalization of knowledge (Hofstetter & Schneuwly, 2017), and the circulation of knowledge within academic networks (Latour, 2000; Waquet, 2022), articulating these perspectives with the historical reading of documentary sources. Methodologically, the study involves the inventorying, cataloguing, and qualitative analysis of correspondence related to the creation of SBHMat, with particular attention to interlocutors, involved institutions, recurring themes, and strategies of scientific articulation evidenced in the documents. As a complementary resource, the software *Gephi* is used for network visualization and analysis, enabling the mapping of connections, flows, and centralities within epistolary exchanges and expanding the interpretative possibilities of the documentary corpus. Although still in progress, the research already highlights the relevance of D'Ambrosio's correspondence for understanding the backstage of SBHMat's creation, revealing challenges, negotiations, and strategies that shaped this process. It is expected that the findings will contribute to strengthening studies on memory, archives, and the institutionalization of knowledge in the field of the History of Mathematics Education, as well as fostering reflections on the constitution of scientific communities in Brazil.

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Past and present in mathematics didactics as a tool in teacher training

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The integration of the history of mathematics into teacher education has been widely discussed and researched in recent years (Arcavi, 2023; Clark, 2019). In contrast, the history of mathematics education–specifically, historical approaches to teaching mathematics and the evolution of curricula and pedagogical ideas–has received comparatively little attention as a resource for teacher training. This study presents an initial attempt to incorporate sources from the history of mathematics education into a mathematics didactics course for prospective teachers. The study draws on a diverse corpus of historical materials, including curricula, textbooks, and teacher reference books from the early twentieth century to the present. These sources were used to design a learning activity focused on a well-known conceptual difficulty in school mathematics: the teaching of negative numbers, with particular emphasis on multiplication.

Methodology

The research was conducted in two stages. First, historical textbooks and curricula were analyzed to identify how the multiplication of negative numbers has been presented over time and to trace changes in instructional approaches.

In the second stage, selected excerpts representing different pedagogical approaches were presented to a group of 34 college students enrolled in a mathematics teacher education program. Working in small groups over a 90-minute session, participants examined three excerpts each. They were asked to evaluate the pedagogical ideas underlying each approach, compare them with current curricula and textbooks, and select and justify the approach they would prefer to use in their own teaching. Participants documented their discussions and conclusions using a structured online questionnaire, which served as the primary data source for this stage of the study.

Findings and Discussion

Analysis of the historical materials revealed three main approaches to teaching the multiplication of negative numbers: extending numerical patterns and properties, preserving the distributive law, and employing models drawn from everyday contexts. The findings from the student responses indicate that the vast majority of participants valued the activity and perceived it as contributing meaningfully to their practical pedagogical knowledge. Students particularly appreciated the opportunity to engage with multiple explanatory frameworks, discuss their merits and limitations, and reflect on their own future instructional choices. Notably, participants expressed comparable interest in approaches originating from different historical periods, with no clear preference for contemporary methods over older ones. This poster reports on a pilot study conducted in December 2025. The positive responses suggest that the history of mathematics education can serve as a productive and engaging resource in teacher training, and the study will be expanded in future research based on the pilot findings.

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Repunit sequence in teaching

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The study of numerical sequences is a significant part of the history of Mathematics, with records dating back to the most ancient civilizations. Evidence can be found in Babylonian clay tablets, dated to approximately 4500 BC, and in the works of the Pythagorean school, between 585 and 500 BC (Eves 2008). In Ancient Greece, the records in Euclid's *Elements*, around 300 BC, stand out (Barrow-Green et al., 2021). A historical account of numerical sequences should be incorporated into mathematics lessons, as history has helped to build a differentiated view of mathematics, which comes to be seen as an intellectual and humanizing activity; it reorients the understanding of the objects of Mathematics and promotes interdisciplinarity, as it establishes relationships with other areas of knowledge, according to D'Ambrosio (2021). Despite their historical and conceptual relevance, numerical sequences are underexplored in basic education, failing to reflect the topic's significance and its formative potential, as noted by Vieira, Alves, and

Catarino (2023). The study of sequences gœs beyond the mechanized approach often employed in basic education, making it desirable to explore this fertile ground for investigation and discovery.

The repunit sequence, as presented by Costa and Carvalho (2024), offers various possibilities to be leveraged in teaching. Repunit numbers can be described as a subset of natural numbers, written by the juxtaposition of the digit 1 (Costa & Santos 2020), that is, numbers such as 11, 1111, etc. A series of mathematical properties and intriguing facts can be explored in the classroom to spark student interest and curiosity, such as questions related to their divisibility, their algebraic representation, and their connections to prime numbers (Carvalho & Costa 2022; Costa & Santos 2022; Costa, Santos & Bezerra 2023). Considering the history of mathematics and numerical sequences, we will plan didactic intervention activities for teaching numerical sequences in mathematics classes, following the methodology of Didactic Engineering. This framework systematically and theoretically integrates pedagogical theory and practice and aims to develop resources for regular teaching or teacher education (Almouloud & Silva 2012). The purpose of this work is to present and analyze the potential of exploring repunit numbers in the mathematics classroom as a valuable resource for engaging students, illustrating algebraic and numerical concepts, and, above all, reclaiming the investigative and humanizing dimension of Mathematics, as advocated by the integration of its history.

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The Influence of Historical Mathematics Content on Elementary Students' Learning Interest: A Focus on Integers

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This study pursued a dual objective. First, it aimed to extract representative historical mathematics materials from relevant books suitable for elementary instruction and to design corresponding teaching activities for classroom implementation. Second, by implementing the lesson plans and conducting classroom observations, the study sought to explore how integrating historical contexts with mathematical knowledge could benefit students' cognitive and affective development. The research involved closely observing changes in students' thinking behaviors during learning and analyzing the effects of integrating the history of mathematics on students' learning interest and potential problem-solving abilities, ultimately providing more diverse instructional strategies or content options for mathematics education.The lesson plans were designed with a practical orientation, following the lesson design principles proposed by the Center for Research on Teaching and Learning at the University of Michigan.The lesson plans comprised a total of five 40-minute lessons. The topics of Lessons 1 to 4 were: (1) Ancient Egyptian Numeral System – Hieroglyphic Script, (2) Ancient Chinese Arithmetic – Counting Rods, (3) Divisibility – Euclid's Elements, and (4) Divisibility – Counting Rods. As a capstone activity(assessment), the fifth lesson engaged students in developing their own numeral systems based on the concepts they had learned. The participants were students from a class at a public elementary school located in an urban area. Based on students' mathematics grades from the previous academic year and two quiz scores from the current semester, we divided the students into three achievement groups – high, medium, and low - achievement groups – by hierarchical cluster analysis. Among the six students in the high-achievement group, two of whom had previously passed the Gifted and Talented identification.

The results showed that students who demonstrated a high level of concentration during class were generally able to understand ancient mathematical methods and compare them with modern approaches. However, students' interests results were somewhat unexpected. Although the six high-achievement students performed the best across all activities , and particularly excelled in developing their own numeral systems, only two students reported that the lessons had increased their interest in mathematics. In contrast, students in the medium- and low-achievement groups proactively asked questions and participated during the lessons, even though they did not perform so good like the high-achievement students. Seven out of nine students in the medium-achievement group stated that the lessons were highly engaging, that they learned more, and enjoyed mathematics more as a result, and in the low-achievement group, two out of five students said that the lessons had increased their interest in mathematics.

Transnational Interactions of Ubiratan D'Ambrosio During the Formulation of the Course Trends in Mathematics Education: The Circulation of Knowledge and the Initial Proposal of This Discipline

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Within the broad theme of teacher education and with a focus on the international circulation of knowledge, this study aims to analyze the transnational interactions established by Ubiratan D'Ambrosio during the formulation of the course TEM (Trends in Mathematics Education) in the mid-1980s, highlighting how such interactions influenced the initial proposal of the discipline. The investigation will be conducted through a bibliographic and documentary approach, based on unpublished materials from the APUA (Ubiratan D'Ambrosio Personal Archive), preserved at CEMAT (Center for Documentation of Scientific and Pedagogical Memory of Mathematics Teaching), located in Santos-SP, Brazil, and managed by GHEMAT-SP (Research Group on the History of Mathematics Education - SP). Preliminary results indicate the existence of correspondence, manuscripts, and records of international events that demonstrate the circulation of ideas in the field of Mathematics Education, allowing for the delineation of a historiography of the creation of the TEM course and its constitutive knowledge. These data show that the first bibliographies suggested for the course, dated from the late 1970s and early 1980s, already incorporated foreign works discussing new methodologies, technologies, and approaches to mathematics teaching, revealing that D'Ambrosio articulated external references to consolidate the proposal of the course in Brazil. Moreover, documents from the APUA show that TEM aimed to prepare teachers capable of understanding mathematics as a historical, cultural, and political construction, in dialogue with international movements such as those of the NCTM (National Council of Teachers of Mathematics), which already advocated the systematic use of calculators and computers in classrooms. Thus, the constitutive knowledge of the course emerges from the intersection between global debates and local practices, configuring TEM as a privileged space for the circulation, appropriation, and re-signification of ideas in the field of Mathematics Education.

Use of historical primary sources to support preservice teachers' insight into mathematical ideas

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3

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Most preservice teachers for primary and lower secondary school in Denmark start their education with high school level mathematics. According to Jankvist's interviews (2008) some high school students "have a certain understanding of the deductive side of mathematics (definition, theorem, proof), but many do not answer the question about the structure of mathematics at all, and some students see no difference between theory construction in mathematics and other natural science subjects" (Jankvist, 2008, p. 43). The Danish teacher education is organized as a professional education in a University College and the goal of the course for preservice teachers in mathematics is among other things, to give them a deep insight into the mathematics that lies behind the school subject, didactical knowledge, and knowledge about the math teachers' praxis in the school (BEK 707, 11/06/2024, Appendix 2). Therefore, we raise the following questions:

How can we as educators support preservice teachers in developing a deeper understanding of chosen topics in mathematics?

How can we strengthen the students' (a) understanding of chosen ideas and theories in mathematics and (b) ability to make didactical choices?

Our hypothesis is that including topics from history of mathematics and primary historical sources can enrich both aspects.

We conducted a project, in which two classes of second year preservice teachers participated. The focus was on proofs and axiomatic systems in mathematics. The two classes studied the definitions and axioms from the first book of Euclid's *Elements* and reproduced chosen constructions. Subsequently, we asked them to write a text describing how working with primary sources influenced their understanding of mathematics, their idea of mathematics as a subject, and their understanding of their ability to do mathematics. We finally coded their answers based on three of Niss' five categories of the nature of mathematics: Mathematics as a scientific discipline, mathematics as an educational subject, and mathematics as an aesthetic experience (Niss, 2001). We plan to continue the project in other two classes focusing on the concept of functions. The preservice teachers will work with excerpts from primary sources and relate the mathematical definitions and their historical evolution with the typical misconceptions pupils have in school. The chosen texts include excerpts from the definition of function given by Euler, Dirichlet and Bourbaki.

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Walking with Archimedes

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The project "Walking with Archimede" aims to explore the historical fi gure and scientifi c contribution of Archimedes of Syracuse and it takes inspiration from reading the book "Il codice perduto di Archimede" by Reviel Netz and William Nœl. The aim is to stimulate interest in mathematics, physics and the history of science, using an interdisciplinary and practical approach. The twinning between parallel classes (two belonging to IC Caponnetto, Bagno a Ripoli, Firenze, one belonging to IC Giuseppe Melodia, Noto, Syracuse) encourages the development of fundamental skills such as teamwork and intercultural communication. The activities of the project have been: meeting between the three classes on the meet platform (October 2024); shared reading in each class of the book "Archimede e le sue macchine da guerra" by L. Novelli; literary café with challenge between classes (on the meet platform) on the contents of the book read;construction of the Stomachion (with the contribution of the Art prof); math lab: games with the Stomachion (geometry); math lab: garden activities to discover; math and physic lab: activities "Levers and proportions"; end-of-year trip to Syracuse (home of Archimedes) for the two classes belonging to IC Caponnetto; room escape and math game between the three classes;theatrical show by the two classes of Caponnetto and Short movie on Archimedes life by the class of Noto; the event took place at the Noto theater; movie on Archimede's life produced by students and teachers of the G. MelodiaInstitute of Noto (Sicily) Visit to the Thecno Park Archimedes and the Greek Theatre in Syracuse.

At the end of the project, students were asked to answer a questionnaire of appreciation.

Theater representation on Archimede's life of the I.C.Caponnetto's students. The event took place at the Noto theater in Sicily: https://youtu.be/H_y2C5MJSnc

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Workshop, 2h

Workshops, based on historical and/or epistemological material (2h), in the program are arranged alphabetically by title.

A Mathematical Toolkit for Middle School, Inspired by Archaeology and History

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We are two middle school mathematics teachers passionate about engaging students in activities situated within historical and cultural contexts, drawing on our respective backgrounds in archaeology and the history of mathematics. Prior to becoming mathematics teachers, one of us was a professional archaeologist, while the other studied the history of mathematics. Convinced that archaeology and history can spark students' curiosity, we have explored ways to integrate these disciplines into mathematics teaching. In this workshop, we will present an educational kit comprising two mathematics activities: one related to archaeology and the other to the history of mathematics. The first activity is based on hands-on manipulations inspired by the work of archaeologists examining artefacts uncovered during excavations. Student groups collaborate to:

1. Reconstruct the spatial distribution of the artefacts (replicas prepared by us);
2. Classify the artefacts according to their location and typology (amphorae, plates, mosaics);
3. Assemble the artefacts to partially reconstruct the objects; and
4. Complete the reconstruction by identifying and applying their axes or centres of symmetry.

The mathematical concepts addressed include signed numbers, plane coordinate systems using (x, y) coordinates, and axial and central symmetries. The second activity is designed to familiarize students with the methodological approach of the history of mathematics. It encourages them to engage with well-known mathematical problems from historical contexts by introducing them to the reading of problems as formulated in primary sources and by exposing them to alternative ways of approaching mathematical concepts. When implementing these activities, we observed strong engagement and motivation among our students. They appeared to grasp certain mathematical concepts more effectively when these were presented in concrete situations connected to engaging archaeological and historical contexts. During this summer school, we will present the activities to participants, allowing them to implement and experience the tasks firsthand. We welcome feedback to explore potential improvements and to learn whether similar initiatives have been conducted elsewhere. This exchange may inform the future extension of our project toward an even more multidisciplinary approach.

Examining the history of mathematics as a Gateway Towards Inclusivity, Engagement, and Understanding in the Mathematics Classroom: Understanding the Irish secondary school context. A report on research in progress.

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There is an emerging body of international research which has explored the role, benefits, and challenges of incorporating the history of mathematics (HoM) into mathematics education. The benefits for student motivation, mathematical understanding, meta-cognitive learning opportunities, and enriching curriculum content with discovery, purpose, and a sense of wonder, are identified. Additionally, HoM gives space for students to engage with mathematics in a diverse means through representation and developing cultural understanding. Teaching mathematics through a lens of history and embedding history learnings in how students engage with mathematics emphasises the humanised aspect of mathematics, critical thinking and problem-solving skills, the nature of mathematics and the qualities of mathematicians.

The Irish context in relation to HoM in mathematics education is generally unreported and, in this presentation, I will provide a report on this research in progress. Specifically, I will discuss my survey of secondary school mathematics teachers in Ireland which will provide a quantitative and qualitative snapshot of how they currently engage with HoM in their classrooms. It will generate an understanding of teachers' current knowledge and training in the area, how they engage with HoM in their classrooms, highlight their views and beliefs on HoM, and the value they place on HoM in the classroom.

To provide added context to these findings, I will provide a brief overview of the current role of HoM in teacher training in Irish higher education. This will be complemented by initial work on a systematic literature review in this area. I will also outline my plans for how these results will aid the development of resources for use in teaching of mathematics in secondary school, but also in the training of mathematics teachers.

Overall, this research is part of a doctoral project *To Examine HoM as a Gateway Towards Inclusivity, Engagement, and Understanding in the Mathematics Classroom*. It seeks to address an identified absence of empirical research in key areas: framing the Irish context of HoM in mathematics classrooms by assessing teacher and student engagement levels with HoM; voicing the perspectives of Irish teachers relating to HoM; leveraging HoM to further democratise STEM and promote diversity and representation; curriculum resource development; and measuring the impact of HoM on student motivation, engagement, and appreciation of mathematics. A mixed methods approach incorporating quantitative and qualitative methods will inform these research areas. A systematic literature review will set the scene in this contemporary space. Surveys and interviews/focus groups will be used to describe stakeholders' perspectives, thus informing the design of curricular materials. These will be preimposed and trialled with in- and pre-service teachers. Given the significant link to the teaching and learning of mathematics, this project will be guided by the Mathematical Knowledge Framework which structures a way to understand mathematical skills, concepts and knowledge. This supports a constructivist approach.

Experimenting the tangibility of Tangent with Historical Mechanisms

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1

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Mechanical machines have played a central role in the history of geometry. They allowed the establishment of many curves and remained the only way to draw them for a very long time, in their very specific continuous way. In the late 17th century, one of the central challenges in mathematics was the geometric legitimization of transcendental curves. Differently from the algebraic case, there was no general method to generalize these constructions, and to justify a new transcendental curve it was necessary to think of a new device tracing it. A promising problem arose in the 1670s aside from mathematics: the physician and architect Claude Perrault proposed the construction of a new curve, the tractrix, whose behavior was defined by the properties set to the tangent to the curve-to-be. That caused a deep rethinking of the meaning of the tangent: the tangent can exist before the curve, and can also define the curve. Such ideas of construction, named "tractional motion," constituted the geometric/mechanical solution to inverse tangent problems. Tractional motion was deepened by scholars like Huygens and the brothers Bernoulli, and constituted the background of Leibniz's geometric conception of calculus. In continuity with such an approach, the Paduan polymath Giovanni Poleni made a notable contribution in the early 18th century by designing and constructing two mechanical instruments capable of continuously tracing the tractrix and logarithmic curves with remarkable precision. Although these devices were once documented in historical catalogues of the Physics Cabinet of the University of Padua, they are no longer present in the collection. We undertook their reconstruction, bringing these machines back into action as functional artefacts of mathematical heritage.

In this exhibition, we present various machines related to tractional motion: Perrault's construction, Poleni's reconstructed machines , two reconstructions of Perks' machine, one of the Jacques Bernoulli's machine, ... and also some devices to draw conics with string or load, in order to explore their similarities and differences. They are shown together with an excerpt of primary and secondary sources, a video presenting the reconstruction of the Poleni's machines and panels guiding visitors through such a tangible approach to the tangent.

A similar selection of material and textual sources was proposed in workshops with mathematics teachers and in a course for third year Bachelor mathematics students to reflect on the geometrical nature of representative curve of some reference functions – like conics, exponential and logarithmic – and to explore the inverse-tangent problem, its historical and epistemological context and implications. This exhibition should foster a rich shift in perspective (from abstract formalism to embodied mathematical experience) highlighting how historical artefacts can serve as powerful tools for didactical reflection and conceptual understanding.

This workshop includes a guided visit to the exhibition alongside its creators.

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Ferrari, Tartaglia, and the Fixed-Opening Compass, or How to Reconstruct the Elements by Changing the Third Postulate

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2

The proposed activity has been designed (and partially implemented) for 11th-grade students. It aims to highlight logical dependence between propositions and postulates, as well as between propositions themselves. The guiding question is what would happen if we modified the Third Postulate of Euclid's *Elements* from "It is possible to describe a circle with any centre and distance" to "It is possible to describe a circle with any centre but only with a fixed distance", so that only a compass with a fixed opening can be used. Would it still be possible to construct Euclidean geometry (within certain limits)? This issue has been raised in the history of mathematics: Niccolò Tartaglia posed this problem to Ludovico Ferrari in the *Cartelli di matematica disfida* of 1547-48 and later proposed his own solution in *General Trattato de Numeri et Misure* (1556-60). Building on these sources, we designed an activity in which historical texts contribute to the construction of the mathematical concept of logical dependence.

The workshop will be structured as follows:

1. Historical and mathematical context. The organizers will present a brief overview of the historical and mathematical context of the challenge between Tartaglia and Ferrari. Then, they will read the problem Tartaglia posed to Ferrari. Participants will be divided into small groups and invited to put themselves in Ferrari's shoes. Ferrari reconstructed all of Euclid's propositions, but the order of the propositions in his version of the *Elements* was different from the traditional Euclidean text. Participants will receive the text of Euclid's postulates and the first propositions, so they can study the direct or indirect dependence on the third postulate and decide which proposition should be the first one in the "fixed-compass version" of the Elements and then which should be the second and third ones. We will limit our analysis to a few selected propositions. This exploration will help us understand the meaning of Ferrari's project.
2. Exploring Ferrari's solution. After the participants have formulated and discussed together their conjectures, there will be a guided reading of some excerpts from the *Cartelli* in which Ferrari explains his "fixed-compass version" of the *Elements*. Each group will be provided with a ruler and a compass that can draw only circles with fixed radius (a rigid compass with a screw to lock the opening). With these mathematical instruments, participants will reproduce Ferrari's constructions. Additionally, the organizers will illustrate a GeoGebra activity in which they will construct the "fixed compass" macro and explore some geometric constructions.
3. Tartaglia's alternative approach. Some passages from the *General Trattato* will be read and analyzed, in which Tartaglia proposed an alternative solution.
4. The workshop will conclude with a plenary discussion comparing Ferrari's and Tartaglia's approaches.

The following materials will be distributed: a dossier containing the postulates and the first ten propositions of the *Elements*, excerpts from the *Cartelli di matematica disfida* (translated into English), excerpts from Tartaglia's *General Trattato de numeri et misure* (translated into English), blank sheets of paper, pencils, rulers and fixed compasses.

Logarithms: A History of Simplification

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3

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At LIFHIMATE (Research and Training Laboratory in History of Mathematics and Education) of CIDMA (Research and Development Centre in Mathematics and Applications) at the University of Aveiro, Portugal we investigate and value learning based on the History of Mathematics (HM) and work with primary and secondary school teachers, promoting their training in HM. This workshop is aimed at secondary school mathematics teachers (students aged between 15 and 18 years old) promoting the integration of HM in conjunction with the current curriculum and with well-founded teaching practices. We have chosen a historical approach to logarithms, deconstructing formal concepts and walking alongside geometers and astronomers on the path to formalization. It is well known that, before the advent of calculators and computers, calculations with large numbers constituted a serious obstacle to scientific progress. Astronomers, cartographers, and navigators needed effective methods to perform multiplications, divisions, and complex trigonometric calculations. The development of logarithms thus emerged as a response to a practical need, culminating in the work of John Napier at the beginning of the 17th century. Although the history of logarithms is a topic already explored and known in mathematical historiography, this workshop traces a path in articulation with the current curriculum and with well-founded teaching practices. The general objective is to understand the genesis of logarithms from real calculation problems and their evolution to the modern notion of logarithmic function, contributing to the conceptual and didactic deepening of the following domains: 10th Grade: The problem of large numbers in Astronomy; Practical activity: astronomical calculations with large numbers (using powers and radicals, understanding operational properties as a conceptual basis for the idea of transforming products into sums), simulation of astronomical calculations with large numbers without a calculator (analysis of the didactic potential of the activity in a classroom context); 11th Grade: Trigonometry as a calculation tool, introduction to astronomical and trigonometric tables, relationship between angles, chords and sines, use of trigonometric identities to simplify calculations, sine tables and their historical role, instruments for measuring inaccessible distances (astrolabe, Jacob's rod); Practical Activity: use of excerpts from historical trigonometric tables and simulation of astronomical calculations without a calculator (Analyze the advantages and limitations of these tools in current teaching); 12th Grade: The fundamental idea of Napier's logarithms (guided reading of excerpts from *Mirifici Logarithmorum Canonis Descriptio*), correspondence between geometric and arithmetic progressions; Simplified examples of Napier's method, formalization of the logarithmic function (Briggs and decimal logarithms, the concept of logarithm before the notion of base, logarithms as inverse functions of exponentials, comparison between the historical approach and the current school approach); Practical activities: 1. Construct a small table of logarithms in Napier's manner, 2. Astronomical calculations using logarithms, 3. Comparison between historical tables and the modern definition of logarithm, 4. Use of the slide rule. (Didactic reflection on the heuristic value of this approach).

As a result, we expect teachers to recognize that: the inclusion of the history of logarithms in mathematics teaching constitutes a relevant teaching resource, as it allows contextualizing the origin and purpose of this concept; logarithms emerged as a response to the need to simplify complex calculations, especially in areas such as astronomy and navigation, highlighting the close relationship between mathematical development and concrete problems of the time; the historical perspective helps students understand the reason for the concept, and not just its technical functioning; the historical study of logarithms promotes the humanization of mathematics by presenting mathematical knowledge as a progressive construction, marked by different approaches, notations, and interpretations; the analysis of logarithmic tables and calculation methods prior to calculators promotes a deeper conceptual understanding and stimulates critical thinking; the proposed approach also constitutes an instrument for pedagogical reflection, enriching teaching practice and fostering more meaningful learning.

Mathematics Across Cultures: Historical Examples as Dialogical Spaces for Understanding, Meaning, and Agency

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2

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This workshop explores the educational potential of historically and culturally situated examples from the history of mathematics as resources for reflective, dialogical mathematics education. Building on a set of teaching examples developed within our ongoing research on culture- and history-responsive mathematics education, the workshop invites participants to actively engage with mathematical problems and practices that emerged in different cultural and historical contexts (e.g. Islamic, European or East Asian traditions).

Through guided work on these examples and structured discussion phases, the workshop pursues two closely related aims. First, it examines to what extent such examples–and the implicit cultural comparisons that arise through their joint exploration–can contribute to mutual understanding, interpretative openness, and the recognition of other cultural traditions. In this sense, historical mathematical tasks are treated as dialogical spaces in which cultural difference becomes a productive resource rather than a barrier.

Second, the workshop investigates how embedding mathematical concepts within their cultural and historical backgrounds can foster deeper conceptual understanding of the mathematical content itself. By situating formal structures, algorithms, or representations within concrete practices, symbolic forms, and epistemic goals of their originating cultures, mathematics is presented not as a culturally neutral body of knowledge, but as a historically grown and socially embedded human activity.

The workshop is designed for researchers, teacher educators, and teachers interested in culturally responsive mathematics education, history of mathematics, and questions of meaning, agency, and identity in mathematics learning. Participants will leave with concrete examples, theoretical reflection points, and didactical perspectives for integrating culturally and historically grounded mathematics into teaching and teacher education contexts.

Mental Representations of Ascending Order around the Plagiograph

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The plagiograph by Sylvester (also called the "skew pantograph") is a generalization of the pantograph, based on a change of perspective, as Sylvester (1875) precisely describes in a paper that will be studied in the workshop. This marks the beginning of a development of kinematic insights that also highschool students can trace. From a cultural-historical perspective, this can be understood as a development of mental representations of ascending order, where first-order representations of operations and objects are closely tied to the hands-on interaction with physical objects derived from reality (Renn & Damerow, 2007). With the embodiments (objects, symbols, visualizations...) of the objects, the same actions can be performed according to certain transformation rules as with the objects themselves. "We abstract empirical knowledge from the objects of our actions and the changes we effect on them and which we attribute to them as immanent possibilities. In contrast, we abstract mathematical knowledge through reflective abstraction from the structures of the coordination of our actions themselves" (Damerow & Schmidt, 2004, p. 134). Historical examples of such first-order representations for the formation of kinematic insights can be seen in the preparatory work for the development of anti-parallelogram, pantograph, plagiograph, Hart's inversor, and the quadroplane. Bars are initially conceived as "arms" and later they represent infinitely large planes and finally mappings. All this provides a basis for understanding the Roberts-Chebyshev Theorem, which allows for finding two "cognates" for any four-bar linkage, i.e., two other four-bar linkages that generate the same curve. This is followed by various inversors and straight-line mechanisms (Sylvester, Kempe's three-kite inversors, and the Kumara–Kampling inversor), and finally, we consider A. Hart's A-frame linkage. We will discover all of this hands-on with real linkages and give proofs in elementary geometry. A paradise for elementary geometry.

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O Homem que Calculava: A Paradoxical Historical Source with Multiple Translations and a Didactic Tool

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O Homem que Calculava [The Man who Counted] is a celebrated book on recreational mathematics, first published in Brazil in 1938. Presented as a translation of an Arabic original, it soon turned out to be a literary hoax forged by the writer and mathematics teacher Jùlio Cásar de Mello e Souza. It is organized as a collection of tales embedded inside a frame story, located in thirteenth-century Bagdad, reminiscent of the style of the *Book of One Thousand and One Nights*. It was translated into some twenty languages–including Arabic, its alleged original language, as late as 2006.

Due to the specific nature of recreational mathematics, where the same patterns travel through times and cultures, *O Homem que Calculava* can and should be considered a primary historical source in its own right, no more or no less than the famous collections gathered by Bachet (1612), Ozanam (1694) or Lucas (1882). Of course, this does not prevent from asking which of the tales compiled are genuinely of Arab-Muslim provenance. In some cases, this turns out to be ultimately the case, but not at the level of the source directly used by the author: although he was fond of Arabic culture–in the general literary context of what has been characterized in 2024 by Waïl S. Hassan as "Brazilian ternary orientalism"–and even took private Arabic lessons with two Lebanese immigrants, Mello e Souza most probably never delved into any complex writing in this language and only resorted to European sources. Some other tales however seem very unlikely to be of Arab-Muslim origin in any sense.

During the workshop, some background information about *O Homem que Calculava* and its sources will be offered. Then, our recent experiments of telling mathematical tales–extracted from that book or similar to it–in the classroom will be described. After that, it will be proposed to set up small groups, working in parallel on different versions of the same excerpts of *O Homem que Calculava*, according to the languages practiced by attendees (mostly the official languages of ESU, i.e. Portuguese, English and French). This will be followed by a plenary discussion about the didactical potential of these excerpts, taking into account the ambiguities inherent to the puzzles, but also the impact of more or less appropriate translations. Where applicable, medieval Arabic or Latin versions of the selected excerpts will also be used. Moreover, participants will be asked to sketch a historical tale for pupils aged between 9 and 12, inspired by the way Mello e Souza used both history and imagination as paradidactic tools to induce a shift of sight on mathematical activities.

Poleni's geometrical machines to construct transcendental curves: a chance to explore the materiality of mathematical concepts today

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In the late 17th century, one of the central challenges in mathematics was the geometric legitimization of transcendental curves. Addressing this issue in the early 18th century, the Paduan polymath Giovanni Poleni offered an innovative contribution. In his *Letter to Hermann*, included in the 1729 *Fasciculus Epistolarum Mathematicarum*, Poleni described two mechanical instruments he designed and built to trace the tractrix and logarithmic curves continuously and with notable precision. Such constructions were based on the mechanical solution of inverse-tangent problems (to find a curve given the properties of its tangents). Although these devices were once recorded in historical catalogues of the Physics Cabinet of the University of Padua, they are no longer preserved in the collection. We have reconstructed them, restoring these machines as functional artefacts of mathematical heritage.

We adopted Poleni's machines in educational activities with mathematics teachers and with third-year university students. Within a broader framework that integrates historical texts and scientific instruments, participants engaged with the reconstructed devices –and few others related– to explore the inverse-tangent problem and its historical and epistemological implications. This work encouraged a significant shift in perspective, moving from abstract formalism toward a more embodied understanding of mathematical ideas. It also showed how historical artefacts can become powerful tools for reflection and conceptual development in mathematics education.

Workshop participants will engage in a comparable experience: they will read selected excerpts from the *Letter to Hermann* while directly handling the reconstructed machines, including the devices for the logarithmic and tractrix curves. The workshop will conclude with a collective exchange of reflections and ideas emerging from these activities, exploring whether the interplay between textual sources and material artefacts can offer fresh insights into inverse-tangent problems from a mathematical perspective, and whether aspects of this experience might be meaningfully transferred into classroom practice.

The workshop will use some of the materials prepared for the exhibition "Experimenting the Tangibility of Tangent with Historical Mechanisms," which will take place during the ESU10.

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Portugal as a playground for geometric fortification during the Restoration War (1640-1668)

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Present day French historians of mathematics know Allain Manesson Mallet, a 17th century military teacher, as the author of the famous book *La Géométrie Pratique* (Paris, 1702), in which many beautiful plates show noblemen measuring distant lengths and drawing geometrical figures on the ground. In his first published book about fortification, *Les Travaux de Mars* (1671), Mallet entitles himself an Engineer of the King of Portugal, appointed Sergeant Major of Artillery in the Province of Alentejo, what led us to focus on Portuguese fortifications, which we unfortunately kind of missed in our initial research (https://theses.hal.science/tel-05360935).

Why Portugal? In 1640 began the Portuguese revolution and King João IV was acclaimed; he had but a few troops and a powerful enemy, that he shared with many other nations, including France. In June 1641, France and Portugal concluded a treaty of alliance, which led Richelieu to favour sending military engineers to Lisbon.

Because indeed, it was a big issue for João IV: to find experts in fortification in order to allow cities along the new border to withstand a siege by the Spanish armies. In Portugal, he had only one real expert, the former Dutch Jesuit priest Jan Ciermans, aka Cosmander. Many French engineers (or so-called engineers) were recruited: Charles Lassart, Michel de Lescolle, Pierre de Massiac, Nicolas de Langres... and Allain Manesson Mallet.

In these times fortification theories were essentially geometrical, dominated by the Dutch School (Stevin, Marolois, Fritach) and the French School (De Ville, Pagan). Geometry was used to conceive the shapes of the fortresses as well as to calculate the lengths and angles of their lines. Moreover, it was used by military engineers in controversies to prove the excellence of their constructions.

The Portuguese School of fortification was initiated in 1647 by Luiz Serrão Pimentel, Chief Engineer of Portugal who taught mathematics and fortification at the first military architecture academy. Pimentel was a severe critic of Manesson Mallet, whom he called an impostor.

So, cruel questions remain: did Manesson Mallet actually conceive fortresses? Did he create plans or copy them?

Workshop in practice : Participants will have to decide whether Manesson Mallet lied or not about his participation in fortifying several places in Portugal (whose names we keep undisclosed yet).

To do this, they will have at their disposal:

- Copies of various maps and plans, including architectural modification projects (completed or not) dating back to the Restoration War;
- Translated extracts of original treatises on fortification (see below), related to the Portuguese places
- Access to online satellite pictures of present-day Portuguese cities, in order to discover what is left from all those beautiful fortresses.

Original texts to be studied with ruler and compass in hand (with English translation)

George Fournier s.j. (transl. Manuel de Villa Real) (1649). *Architetura militar ó fortificacion moderna*. Paris: Jean Henault.

Allain Manesson-Mallet (1671). *Les Travaux de Mars ou la fortification nouvelle tant régulière, qu'irrégulière*. Paris: Jean Henault.

Luiz Serrão Pimentel (1680), *Méthodo Lusitânico de desenhar as Fortificações das Praças Regulares e Irregulares*, Lisbon : Antonio Craesbeeck de Mello.

Reviving Forgotten Mathematics: The Case of Gaspar Nicolas's *Tratado da Pratica d'Arismetica*

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In this workshop, we will follow the methodology used in Moyon (2024). We will first introduce and contextualize the major work of the Portuguese author Gaspar Nicolas, *Tratado da Pratica d'Arismetica* (Clain, 2020), first published in 1519. This text, written in the early sixteenth century, will be presented within the broader scientific and cultural context of Portugal at the time, a period marked by the rise of humanism, the expansion of commerce and navigation, and the growing importance of practical mathematics for education and professional life. We will highlight the specific role played by arithmetic treatises in shaping mathematical knowledge and practices.

The second part of the workshop will invite participants to engage directly with a selection of problems from the treatise. These problems, chosen for their thematic variety, illustrate the range and richness of the work. They include examples grounded in practical geometry, problems involving arithmetic progressions that reveal the development of early algebraic reasoning, and many trade affairs problems (notice that some of the problems were real but others are only recreational to play with the mathematics by itself). Working through these problems together will allow participants to appreciate both their historical significance and their potential relevance for mathematics education today.

Finally, we will open a discussion on the pedagogical potential of these historical problems in contemporary classrooms, as in Clain & Slisko (2025), and/or in teacher training using Moyon (2022) and Martins et al. (2022). Drawing on experiences conducted in Portugal, where Gaspar Nicolas is a relatively well-known figure, and contrasting them with the situation in France, where his name remains unfamiliar to most, we will reflect on how such historical material can enrich mathematics teaching. Particular emphasis will be placed on how engaging with these problems fosters critical thinking, highlights the cultural dimension of mathematics, and offers teachers and future teachers a broader perspective on the diversity of mathematical practices across time and space.

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Sobre a sequência e as extensões de Leonardo e os números figurados gregos: propriedades e sua interpretação para sala de aula por meio de Tabuleiro e ladrilhos

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A pesquisa em Matemática Pura sobre sequências numéricas recorrentes costuma dedicar esforços na elaboração e descoberta de novas classes de sequências, pela generalização dos modelos já existentes, bem como, pela aplicação de suas propriedades em outras áreas da Matemática. Contatamos muitos casos de sequências numéricas que, depois de algum tempo, suas extensões e propriedades generalizadas adquiriram enorme vigor, sobretudo, depois da década de 60. De modo especial, quando consideramos o caso emblematico da Sequência de Fibonacci, passados alguns séculos, foram introduzidas na literatura cientifica propriedades sobre a sequencia Tribonacci, Tetranacci, Pentanacci, etc.

A partir desse exemplo, na presente proposta de workshop abordaremos os casos da sequência de Leonardo e propriedades de suas extensões e propriedades dos números figurados gregos, com interpretação inédita correspondente via Tabuleiro com ladrilhos. Muitas propriedades matemáticas desses dois objetos matemáticos podem ser descritas e interpretadas por intermédio da visualização e de forma heurística, a partir de um cenário de aprendizagem significativo e, além disso, com uma respectiva interpretação via o Tabuleiro do software GeoGebra. Assim, a partir da interação e da manipulação de propriedades artimeticas e combinatórias, os participantes podem aquirir uma incremento de uma cultura histórico-evolutiva sobre tais sequências numéricas.

Finalmente, na presente proposta de WORKSHOP evidenciaremos aos participantes que o conhecimento matemática continua em pleno desenvolvimento e, por intermédio dos casos abordados, demonstraremos aos participantes as possibilidades de seu ensino e abordagem metodológicas para o contexto escolar e visando o aperfeiçoamento da atividade do professor de Matemática em atividades de exploração da História da Matemática em sala de aula.

Solving problems from the *Arithmetica Practica, y Speculativa* by Juan Pérez de Moya (1562), in secondary education

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Working on ancient mathematical texts with students helps them understand the nature of mathematics, a science in continuous evolution in which problems linked to the interests of each era are solved, whether they are problems of everyday life, related to the world around us, of a more speculative nature, or why not? recreational.

One of the goals of the ABEAM history group, to which we belong, is to design activities for learning mathematical concepts and procedures from original sources. In this workshop, we will demonstrate how a 16th-century text, Book Nine of Juan Pérez de Moya's *Arithmetica practica y speculativa* (1562), can be used for this purpose, to compulsory secondary education (12-16). We will begin with a brief presentation of the author and his work and show some of the problems from this ninth chapter, as well as an example of an activity designed from one of these problems. Then, working in groups, participants will adapt other problems from *Arithmetica practica y speculativa* into classroom activities. The session will conclude with a group discussion, the conclusions of which we will collect and send to any participants who wish to receive them.

The statements of the problems from the original text by Juan Pérez de Moya, the transcription into Spanish and the translation into English will be distributed among the attendees to facilitate the work in the workshop.

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Teaching the impossible: squaring the circle in 19th-century geometry textbooks

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In the 19th century, rapidly developing educational systems required the production of an ever-increasing number of textbooks, particularly in geometry. At that time, the question of squaring the circle was a major topic of discussion. As early as 1775, however, the Academy of Sciences decided it would no longer review papers dealing with this problem–more than a hundred years before its mathematical resolution. It was Ferdinand von Lindemann who finally settled the question in 1882 by demonstrating the transcendence of π, thereby proving the impossibility of squaring the circle.

Between these two dates, authors of geometry textbooks presented their knowledge on the subject in a variety of ways, depending on their intended audience and their own professional and scientific backgrounds. Some sought to explain this impossibility in different ways, often referring to the nature of π, and connecting (or not) the geometric question to the numerical problem, according to their contemporary understanding of numbers and magnitudes.

In this workshop, we propose to read and analyse excerpts from 19th-century textbooks addressing this problem. These sources should enable us to examine how their authors approach the teaching of questions considered impossible, and to highlight the complex relationship between scientific production and the construction of knowledge to be taught.

These issues can be related to several aspects of current mathematics education in France. Curricula emphasize scientific and historical culture starting in Cycle 3 (8-10 years old). In this regard, the question of squaring the circle can serve as a starting point for reflection on the relationship between geometry and numbers, on the notion of magnitude, and more generally on the scientific method, particularly in relation to impossible problems.

In high school, the problem of squaring the circle can be an opportunity to discuss the nature of numbers, especially that of ?, but also their geometric constructability. It also provides a context for addressing the nature of proof, the validation of results, and the concept of rigor. Squaring the circle is part of the "area calculation" theme in the "complementary mathematics" option in the final year of high school, and in "science teaching" programs, which emphasize the connection between science, culture, and philosophy. Finally, all these elements can be used in teacher training as a way to step back and reflect on future teaching practices.

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The Making of the Greatest Maths Book in the World: A workshop on bringing episodes in the history of mathematics to life in the classroom by means of theatre, incorporating a sequence of short playlets set in ancient Greece

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This workshop (following those presented at ESU-8 Oslo, BSHM/CSHPM (2021), and ESU-9 Salerno), provides an opportunity to experience and reflect on the ways that the devices of dialogue and drama can bring mathematical ideas and history to life in the classroom. We will demonstrate the power of lively and (almost) spontaneous theatre, involving all participants in the production and enactment of a series of very short pre-scripted playlets. We will then reflect together on what we have been part of, share any similar experiences of theatre, and discuss how these might be used to enliven classroom teaching.

The plays evoke the context within which Euclidean geometry arose, and the very idea of mathematics as a deductive science. Continuity and commentary is provided by a narrator, Maria, and a modern mathematics teacher, Emmy.

1. Play (1) Mathematics is All: The Pythagoreans. Maria, Emmy, Proclus.
2. Play (2) Mathematics is Within You: Socrates & the Boy (based on Plato's *Meno*). Socrates, Meno, Boy.
3. Play (3) Mathematics is the Bridge: Plato and his Students. Maria, Emmy, Plato, Eudoxus, Proclus, Thomas Heath, Aristotle.
4. Play (4) No Royal Road: King Ptolemy meets Euclid. Ptolemy I Soter, Euclid, Attendants.
5. Play (5) The Greatest Maths Book Ever Written; Maria, Emmy, Thomas Heath, Girolamo Cardano, John Aubrey, Isaac Barrow, Abraham Lincoln, Bertrand Russell, Bartel van der Waerden, Albert Einstein.
6. Play (6) The Most Beautiful Thing in Mathematics: Johannes Kepler, Philippus, Euclid. The young Euclid engages with his teacher on the sacred rectangle and the sacred pentagram, wondering at the beauty and symmetries of the five regular solids, and being inducted into the geometry that he would later systematize and present as Euclid's *Elements*.

Designed to capture the excitement of this formative period in mathematics, the dialogue draws from primary sources, and is accessible and engaging, spiced with humour and emotion. The play cycle displayed in this workshop aims to motivate the learning and teaching of geometry and inspire interest in authentic contextual history of geometry. It also aims to exorcise the fear of the idea of 'mathematical proof' and plant the seeds of a love for the deductive purity of mathematics that distinguishes it from all else.

Synopsis: Introduction and allocation of parts for all – actors, directors, stage/light/sound personnel; distributing highlighted scripts, items of dress and props (15 minutes). Preparation of stage and rehearsal with directors in separate corners (15 minutes). The show goes on (45 minutes). Plenary session sharing immediate reactions and discussing how the plays might be used and improved for the classroom at various levels (10 minutes). Discussion in smaller groups with worksheets (20 minutes). This may focus on how such an experience of the ancient Greek vision of mathematics might give fresh inspiration for teaching and learning; also, on the potential use of theatre in participants' own teaching, and on how to use primary sources and biographical material to construct appropriate plays or dialogues. Conclusion with group reports and final remarks (15 minutes). This workshop will thus lead participants through the cycle: enact, critique, reflect, create!

The Royal Gymnasium of Lissa and its Jewish mathematics teacher Julius Toeplitz

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With the Second Partition of Poland in 1793 and the subsequent establishment of the Prussian Province of Posen in 1815, the Prussian state came to rule over a territory that was religiously and ethnically more diverse than any other within the kingdom. In 1815, the province had a population of approximately 800,000 inhabitants (about 8% of Prussia's total population). It largely consisted of Polish-speaking Catholics (around 500,000, representing 35% of all Poles in Prussia) and was thus the only Prussian territory in which Germans formed a minority. In addition, Posen was home to roughly 50,000 Jews (about 40% of all Jews in Prussia), making it the only province with a substantial Jewish population. The Prussian authorities responded to the unwanted diversity within their newly acquired territory with policies of discrimination and forced assimilation, albeit with significant variations over time and between the two minorities.

Processes of Germanization, as well as the consolidation of a Polish national identity and the gradual emancipation of the Jewish population, can be closely examined through individual educational institutions. One of the initially three (later more numerous) secondary schools in the province was the Reformist Gymnasium of the town of Leszno/Lissa, which was reopened in 1821 under state supervision as the Royal Gymnasium of Lissa. Its history is closely intertwined with that of the town of Lissa, already known in the Polish period as a "city of different faiths." The annual school reports demonstrate the school's multiethnic and multireligious self-conception, as upheld by its director, teachers, and pupils, while administrative documents reveal the discontent of the Prussian Ministry of Culture and local school authorities with this situation.

One of the most interesting figures during the period under consideration is the Jewish teacher of mathematics and physics Julius Toeplitz (1825-1897), grandfather of the mathematician Otto Toeplitz. Educated at the Royal Gymnasium of Lissa and the University of Breslau, Julius Toeplitz took advantage of the brief window of opportunity opened by the revolutionary events of 1848 to complete his probationary year at the Royal Gymnasium in Lissa – an opportunity that had long been denied to Jews before and would remain inaccessible for many years thereafter. Although highly regarded by the school director, respected by his colleagues, and popular among his pupils, official antisemitism within the Kingdom of Prussia prevented his permanent appointment for more than two decades. Internal documents not only trace this prolonged struggle with state authorities but also illuminate a remarkable episode of Jewish emancipation in the Province of Posen during the nineteenth century. At the same time, his career reflects the broader conditions of secondary education in a province governed by Prussia but largely shaped by a Polish-speaking population.

Women's Voices in the History of Mathematics Education: Learning with Agnesi, Everest Boole, and Chisholm Young

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This workshop presents three proposals inspired by the works of women scientists– Maria Gaetana Agnesi (1718-1799), Mary Everest Boole (1832-1916), and Grace Chisholm Young (1868-1944) –each addressing a different educational level: primary, lower secondary, and upper secondary school. Covering the eighteenth to early twentieth centuries, these activities illustrate how the history of mathematics can enrich teaching and learning at all stages. Exploring these authors' writings offers insight into both the development of mathematics education and the role of women in shaping mathematical knowledge. From a didactical perspective, their work helps restore visibility to often-overlooked figures; presenting them as thinkers and educators also provides meaningful role models, inspiring curiosity, creativity, and confidence in science.

The workshop will be structured in three parts, from primary to upper secondary school. In each part, participants will work with excerpts from the original sources and engage in hands-on activities followed by discussion, reflecting both on the mathematical content and on its pedagogical and historical implications.

1. Maria Gaetana Agnesi was the first woman mathematician to write an analytical treatise with a clear pedagogical purpose (Agnesi 1748). Written in Italian to make it accessible to a wider audience ("la gioventù italiana" -

the Italian youth), the work was intended to guide students clearly and simply through the study of algebraic, geometric, and analytical methods. Agnesi's textbook represented a significant example of the dissemination of the Leibnizian approach to infinitesimal calculus in Italy. Intended for secondary-school students, Agnesi's treatise can still be read in class to introduce the first elements of calculus. The workshop will present possible approaches to the analytical study of curves and the determination of tangents.

2. Mary Everest Boole focused on nurturing children's mathematical imagination as a foundation for scientific thinking, claiming that observation, narrations and hands-on experience enhance the power of the child's mind. She used imaginative stories to reveal the roots and the core ideas of mathematics and some of her lessons, described in the book *Lectures on the Logic of Arithmetics* (1903), are filled with vivid mathematical dialogues. We will reread a story from the mentioned book in a contemporary key, to propose a hands-on activity on the greatest common measure. The author stressed the geometrical nature of the procedure, and went straight to the mathematical meaning, while in everyday textbooks this topic considered a mere arithmetical procedure, in a way that the technique hides the actual meaning. The hands-on activity will reproduce the idea traced by Everest Boole in the sample lesson about the GCM.
3. The work (Young & Young 1905), mainly due to Grace Chisholm Young, fits within the broader field of "on the foundations of geometry, with a particular focus on elementary mathematics from a higher standpoint. The book aimed to provide students with the tools needed to develop "a useful and reliable geometric sense, "serving as an excellent guide for anyone wishing to teach children the foundations of geometry–not only its notions but also the habits of observation and scientific thinking, highlighting the connection between geometry and everyday life. The pedagogical methods required only paper, pencil, pins, and occasionally scissors, and made use of explanations accompanied by "proofs by folding." Another noteworthy aspect is the idea of presenting plane and solid geometry simultaneously. The workshop will illustrate several two- and three-dimensional examples, such as the definition of the square and the cube, and the procedure for folding a square and a cube.

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Workshop, 1.5h

Workshops, based on didactical-pedagogical material (1,5h), in the program are arranged alphabetically by title.

A C-OW and four Pouillys: a variety of uses of local resources from elementary school to teacher training

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2

The current C-OW project is a sequel of the former CorMéCoULi project, which focused on creating classroom activities based on medieval accounts of the Loire Valley region in France (https://www.univ-irem.fr/mathematiques-et-comptabilite-medievale-une-mallette-pedagogique in French). As our IREM in Dijon was involved in the initial project, we thought it was a shame that no such project existed in Burgundy: the idea of C-OW was born. Let's explain first the acronym: the capital letter C refers to the initial project and OW means "outside the walls". These walls are geographical as well as time-related. The task we proposed to our colleagues was the construction of classroom activities based on original documents from Burgundy, without being limited to medieval accounting, even if computing with tokens remained one of the major themes of C-OW. These colleagues were all new to the use of original documents in the classroom. Our main purpose was to question the methods of creation, implementation and evaluation of the activities thus constructed. Indeed, many years of experimentation made at the IREM in Dijon have shown us that there is no single and infallible protocol for the use of original texts in the classroom, and that the choices remain open as regards the way to adapt (or not) the documents, their presentation, the selection of appropriate content for the students, etc. The texts we work on in 2024-2026 with primary school teachers (cycle 3) and middle school teachers, as well as to pre-service mathematics teachers, are from various periods and deal with various subjects:

- The accounts of the Duchy of Burgundy from 1350 to 1550 contain details of financial levies (taxes and duties) and in-kind contributions (grain, wax, etc.) rigorously recorded by the Dukes' accountants. It should be noted that the numbers are written in Roman numerals and that the units of measurement were forgotten at the end of the Ancien Regime.
- The likely arrival of King Charles VI in Burgundy at the end of the fourteenth century was the subject of supply estimates for the places crossed by the Royal Court, with detailed number and prices of the foodstuffs to be purchased and the equipment to be supplied. As the original document has disappeared, only a eighteenth century copy of it is preserved, but it is riddled with errors to be corrected...
- The archives of the archbishopric of Sens contain a measurement of a load of stones blocks for the enlargement of the cathedral, dating back to 1505. The dimensions of the stone blocks are given in feet and inches, and the volume of ninety-seven blocks is given without explanation, but the results are sometimes incorrect.
- The acquisition by the President of the Parliament of Burgundy of land in the village of Pouilly near Dijon at the beginning of the 17th century required the drawing of several plans by the surveyor Royhier. Comparing these plans to current satellite images makes it possible to search for limits of the close of the former castle of Pouilly, which has now become a simple district of Dijon.

By the way, it is one of the four Pouillys which give the title of the workshop. The other three are to be discovered on the spot! Participants will be presented (and they will have to study) the original texts, as well as their transformation into didactical documents, and the progress report of our C-OW project.

An escape room on ancient Indian combinatorics

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2

In this workshop you will play an escape room that is based on the history of mathematics. In my professional opinion and experiences with students at different levels (including teacher training programs), history of mathematics and escape rooms combine rather well. The history of mathematics has a lot of stories to tell and puzzles to solve. The theme of the escape room we will play in this workshop is ancient Indian combinatorics. Players will solve some authentic combinatorics problems, some of which date back to several centuries BCE.

The workshop consists of three parts: playing the game, reflecting on it and identifying design principles for creating such games. First, after a short introduction, participants will form groups and start playing. They will receive an envelope which contains the story, the puzzles and a collection of tangible game materials. After finishing the game, participants will be asked to describe their experiences individually. We will then discuss as a group both content and form of the escape room. What were the learning objectives? What did you think of the puzzles and the story line? Do you believe this type of activity could be useful in your own classroom? On what level(s) and with what contents? Finally, we will end the workshop with some helpful designing principles for creating your own escape room using the history of mathematics.

The use of the escape room format can be especially interesting for teacher training programs, since it also naturally provides the possibility of discussing elements of game-based pedagogy and maker-education in mathematics education.

Analog Tools Before the Digital Age: Rediscovering Historical Geometry Instruments in the Classroom

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1

The workshop focuses on several historical measuring devices. Apart from the polar planimeter (Klein, 2004) – a nearly forgotten analogue instrument once used for determining the area of plane figures – the workshop briefly introduces so-called variable scale (Gerber, 1953) and an instrument originating in 16th century – sometimes considered to be an evolution of a standard compass – called a sector (Tomash and Williams, 2003). The principle of these devices is described briefly (using elementary approaches as, for instance, described by Henrici (1894)) and several examples of classroom activities offered, in which the devices could be employed. One such instance is the use of polar planimeter for teaching integral calculus or using the other two devices as an introduction to line segment subdivisions at primary schools.

The discussion is framed by reflections on the role of technology in mathematics education. While modern digital tools make computation or measuring effortless, they often obscure underlying mathematical ideas. Reintroducing historical or analogue instruments such as the planimeter or the sector can thus create moments of curiosity and discovery–demonstrating that even simple technologies may evoke a sense of wonder while compared to modern digital tools. Furthermore, the stories behind invention of these tools might guide the classroom activities. The instrument also demonstrates the ingenuity of people in times before such problems began to be solved using computers. Although these tools are rarely encountered today, activities involving them can enrich mathematics lessons, offering potential for many didactically sound tasks. Even though such tools might not be commonly available nowadays, there are various online resources showing how to construct them using 3D printing or common building sets (such as LEGO) or everyday objects.

After the interactive introduction of these devices and their demonstration, the participants construct or are offered sector devices from the materials provided. Using these, they learn to solve several basic problems. The workshop concludes with a plenary discussion of possible activities, benefits and obstacles these devices might bring to classrooms.

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Arithmetic with Papy's minicomputer

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2

Contrary to what its name suggests, Papy's minicomputer is not an electronic device but a teaching tool that can be easily crafted from a cupboard or that can be even drawn on paper. Papy's minicomputer is a two-dimensional abacus that combines the decimal positional system to write numbers with the binary number system. Georges Papy, the leading figure of the modern mathematics or New Math movement from the late 1950s till the early 1970s in Belgium, constructed his minicomputer to allow young children to easily perform calculations with large numbers. Papy's minicomputer was extensively used and promoted by, amongst others, Frédérique Papy-Lenger, Papy's wife, for example in the Comprehensive School Mathematics Program (1978).

In this workshop, we will first present the origin and functioning of Papy's minicomputer. Then, participants will be challenged to devise their own algorithms for basic arithmetic operations, addition, subtraction, multiplication and division, or more advanced calculations, i.e. calculating powers, square roots and logarithms (Socas Robayna and Espinel Febles , 1989), using Papy's minicomputer. Playing a set of Minicomputer Golf or Tug of War is also part of the possible activities. Due to the variety of possible activities, the workshop is suitable for primary and secondary school teachers, teacher trainers, and students in a teacher training program.

We first considered Papy's minicomputer from a historical perspective in Gœmans and De Bock (2025). However, Alexandra Sofia Rodrigues brought to our attention that Papy's minicomputer is still used today in some Portuguese educational institutions (Soares Silva Teixeira, 2017). Therefore, we would love to meet teachers, students or researchers who have experience using Papy's minicomputer as a teaching tool.

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Binary System and Boolean Algebra: A Teaching Journey through History and Manipulation

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This workshop presents a practical, interdisciplinary, and inclusive pedagogical approach to teaching the binary system. The methodology is built upon two synergistic pillars: historical research and hands-on manipulation. In this workshop, teachers will be presented with a classroom path designed to make abstract mathematical concepts tangible and accessible to all students, including those with Special Educational Needs (SEN). The journey begins with the physical construction of number bases using interlocking cubes. This tactile experience solidifies the concepts of positional notation and powers, transforming abstract logic into a sensory reality. To support diverse learning needs, specific multisensory tools will be introduced, such as a "10-square stamp" for students with dysgraphia and tactile aids for the visually impaired. The workshop then transitions to historical inquiry, exploring the origins of binary logic through a multicultural lens–from the ancient Chinese I-Ching to the foundational works of George Boole. By integrating historical narratives and recreational mathematics (such as Lucas's Fan), the workshop follows a pathway which provides a significant emotional and cultural context. This approach not only fosters active engagement among "fragile" students but also highlights the global evolution of mathematical thought. The workshop is divided into modules which are designed to be adaptable to different classroom levels. Using interlocking cubes and specialized stamps we can visualize "numerosity" and manipulate the theoretical pillars of the binary system. We provide guidelines to design a mechanical counter using both 3D printing (technology integration) and recycled materials (promoting the "5 Rs" of Civic Education). We show practical exercises using the manufactured materials to convert between binary and decimal systems. We connect this work with the primary sources by reading excerpts from The Mathematical Analysis of Logic (1847). We compare the four basic operations in binary and decimal systems, sparking a debate on "Man vs. Machine" and the relative advantages of different bases. We present a precursor to binary logic, the I-Ching (an ancient wisdom text from 300 BC), and an historical challenge, Edouard Lucas's game in which participants use guided questions to uncover the Boolean principles that ensure the game's unique solutions. Finally, we share customizable assessment rubrics, tests, and questionnaires to help teachers implement and evaluate this journey in their own classrooms.

In summary, this is a 1.5-hour workshop targeted to Middle and High School Teachers. For this workshop we will distribute texts and worksheets for teachers and students to use in the classroom, including a list of all the material and tools used. For the activities envisaged in the workshop, for the duration of the workshop we will supply 10 square stamps, Interlocking Cubes, Edouard Lucas's game and the I-Ching text.

Bringing the History of Mathematics into the Classroom

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2

This workshop demonstrates how teachers can meaningfully integrate historical and multicultural perspectives into mathematics lessons to enhance students' engagement and sense of mathematical belonging. It builds on a seven-week classroom intervention conducted with Year 7 and 8 students (ages 11–13) in an English secondary school, in which historical content was incorporated into regular mathematics lessons. Quantitative results from pre- and post-intervention questionnaires indicated significant positive changes in students' enjoyment of mathematics and their engagement with historical material. Participants will engage directly with the materials used in this intervention and related teacher-training activities. These include:

- Texts and visual sources illustrating historical mathematical practices (e.g. excerpts from the Ahmes Papyrus, *Liber Abaci*, and the *Nine Chapters on the Mathematical Art*; diagrams from Euclid).
- Classroom worksheets and teacher notes designed to link these sources to curriculum topics such as fractions, ratios, indices, and number systems.
- Sample data from the Milton Keynes experiment demonstrating how historical content affects students' affective domains (motivation, anxiety, cultural awareness).
- A lesson-design template to help participants adapt historical material to their own contexts.

The target audience comprises in-service and pre-service secondary-level mathematics teachers, teacher educators, and others interested in the educational uses of the history of mathematics. However, many of the examples can be adapted for upper-primary or senior-secondary levels.

The 90-minute session will be structured as follows:

1. Introduction (20 min): brief presentation of the research background and pedagogical rationale; overview of materials.
2. Group work (40 min): participants, working in small groups, will examine selected historical sources and discuss:
   - What mathematics is involved?
   - Which curriculum topics could it support?
   - How might it be used in a lesson (starter, extension, project, discussion)? Each group will complete a short planning sheet outlining a possible classroom activity.
3. Plenary discussion (20 min): sharing of ideas, reflection on challenges, and synthesis of principles for integrating history effectively.
4. Conclusion (10 min): reflection on broader epistemological and multicultural dimensions of mathematics, and invitation to continue collaboration via the *History and Mathematics in Education Network* (`https://historyand.mathsy.space`).

Participants will receive all handouts in print and digital form; optional pre-reading materials (including historical excerpts and sample lesson plans) will be posted on the ESU-10 website before the event.

The workshop aims to equip teachers with both conceptual understanding and practical tools to humanise mathematics, diversify its narratives, and enrich classroom experiences through authentic historical content..

Comparing versions of sources as part of a master course in history of mathematics for teacher students

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3

In a master course on history of mathematics for pre-service and in-service teachers for grades 1-10 (age 5-16), we often give students different versions of the same or similar text, to analyse for similarities and differences. For instance, we look at

- Propositions from Euclid's *Elements*, including a version close to Heath's version, Barrow's version and Byrne's colourful version (and perhaps a version using dynamic geometry in addition).
- Propositions from Diophant's *Arithmetica*, both more literal translations (Mendell, Drabkin) and more summary translations (Heath, Katz).
- A fraction of the Bakhshali manuscript, in original, as well as a translation (Kaye) and a modern summary (Joseph).

In the workshop, we will work in groups looking at some of these texts, and have the same kind of discussions that we have with our students, on questions such as:

- What are the key differences between different versions?
- What can we learn from reading a version as close to the original as possible – and what can we learn from

reading a more pedagogically edited version? (perhaps considering Grattan-Guinness' concepts of history vs. heritage)

- What dœs Barrow's version tell us about mathematics at the time of Euclid – and what dœs it tell us about mathematics at the time of Barrow?
- Different versions are written for different purposes – how should our purpose in reading them influence which versions we choose to work on? Which versions would we give students in school? (our students' read some key texts on the purposes of including history of mathematics in mathematics education, which is a useful background to these discussions)
- Can we find examples where reading a particular version that may give a risk of misconceptions of the history of mathematics?

Of course, we will also discuss the use of different versions of texts from the history of mathematics in teacher education – what points can we get across using multiple versions that are more difficult to get across using singular sources?

The course in question is taught at Volda University College by myself and my colleague Hilde Opsal (who can not attend the ESU). The examples used are from my part of the course. The workshop will mostly be done by working in small groups, with just a short introduction and a little summary at the end.

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Criteria for the didactical transposition of historical mathematical knowledge: Their analysis, relevance, and applicability in the case of solving cubic equations and introducing the imaginary numbers

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1

In recent work ([14 - 16]) we stressed the need for the didactical transposition ([5]) of historical mathematical knowledge (an idea also put forward earlier by Kjedsen [12]), in a way that achieves a subtle balance by staying faithful to historical knowledge, while adapting this knowledge so as to fit the needs and educational level of the learners; evidently a highly nontrivial task as has been thoroughly explained by Fried ([6, 7]).

To this end we also gave a tentative formulation of a possible framework in the form of four criteria to be satisfied by teaching proposals and we illustrated them by means of particular examples from secondary education algebra and geometry ([15, 16]). These criteria should be seen as a whole of interconnected requirements adaptable to different educational systems or levels of historical understanding, amenable to modifications that will emerge via its application to specific teaching proposals. And conversely, these criteria could help to analyze important proposals that have been formulated in the past for didactically using the history of mathematics and which aimed to provide answers to didactically-triggered questions (e.g. Why is a given concept or method introduced? How is it properly treated in a mathematically meaningful way? Why dœs it cause so many misconceptions? etc.).

Along these lines, in the proposed WS we intend first to give an outline of specific proposals put forward in the past (as teaching proposals, or/and implemented teaching approaches) concerning the solution of cubic equations and the introduction of complex numbers ([1-4, 8-11, 13]). Then we will discuss the four criteria mentioned above in relation to these proposals in order to consider their compliance with these criteria and conversely, to examine whether and how these criteria need to be refined in view of or/and motivated by the analysis of this particular historical case.

The papers to be discussed will be made available to prospective attendants in advance via the ESU- 10 website and excerpts from them will be distributed on the spot for easy reference during the WS. We will encourage attendants to make their point in relation to the significance, conciseness and adequacy of the four criteria (by considering one or more of the works above according to their preferences) and to propose eventual modifications they think necessary, and also invite them to suggest other works or publications on the subject that could be discussed (this invitation will be posted on the ESU-10 together with the excerpts of the works suggested for discussion, so that the material to be discussed in the WS will possibly be enriched beforehand).

From a mathematical point of view, the subject matter is at the level of upper secondary education mathematics curriculum, but of course, the discussion in the WS will hopefully be useful and insightful for those interested in teacher education.

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Exploring Historical-Mathematical Tasks: From Textbooks to Classroom Practice

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This workshop presents the research carried out by the group "History of Mathematics and Textbooks" at the IREM of Limoges (France), which brings together university researchers and high school mathematics teachers (working with students aged 15 to 18).

Following Moyon (2021), our objective has been to propose a new categorization of historical-mathematical tasks drawn from school textbooks, based on the typology developed by S. Schorcht (2018a, 2018b). The aim of this categorization is to provide a more precise framework for describing and analyzing the different types of tasks that may be integrated into classroom practice, considering that textbooks remain a central resource for teachers and students. It offers the possibility of rethinking these tasks by articulating, in a coherent way, their mathematical, historical, and didactical dimensions. After developing this categorization, we conducted a classroom experimentation phase in order to assess its relevance. Selected tasks were implemented with students, whose written productions were subsequently analyzed. In parallel, an individual and anonymous questionnaire collected students' perceptions of the variety of activities proposed. The cross-analysis of students' productions and questionnaire responses constitutes the basis of our results.

During the workshop, we will first present the categorization by highlighting the principles that guided its elaboration. Participants will then be invited to work on several examples taken from French textbooks (translated into English) in order to gain a deeper understanding of the approach and its potential benefits. In a second step, we will share the outcomes of classroom experiments conducted with 106 students (aged 16-18), focusing on the conditions of implementation, the students' productions, and their responses to the questionnaires.

Finally, we will open a discussion on the contributions and perspectives of this categorization for mathematics teaching and for the integration of historical approaches in the classroom.

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Getting into mathematical, historical and pedagogical questions through video-clips (echoing Bernard's plenary lecture)

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This workshop complements Alain Bernard's plenary conference, "History of Mathematics for Future Teachers, in a Nutshell." A sample of two different videos from the pedagogical video clips mentioned in the lecture will be presented in some detail. The series, entitled "histoires de mathématiques et de techniques," is intended as a pedagogical tool for both pre-service and in-service teacher training courses.

Following the general format used throughout the series, each clip consists of four distinct scenes, performed by two characters–one of whom represents a teacher with limited expertise in the history of science. The introductory scene presents a highlight object, such as a mathematical instrument, which serves as the central focus of the discussion. It also introduces various issues that are developed in the rest of the clip and concludes with the episode's title.

This is followed by three different scenes, each one of them concluded with questions to be studied and answered collectively. The first concludes with a simple mathematical question related to one aspect of the highlight object. The second ends with a general historical question on the same and introduces an accessible online resource to facilitate the search for an answer. The final scene presents a dispute–a dilemma posed by the two protagonists–and invariably ends with the question "et vous, qu'en pensez-vous?" that is: "And you, what do you think about this?"

The aim of the workshop is not only to explore the selected clips through preliminary brainstorming on the questions, but also to discuss their potential from various perspectives:

- As pedagogical tools: What concrete uses can be made of them? For what purposes and within which pedagogical scenarios? For what kinds of trainee teachers?
- As models for similar videos or other pedagogical scenarios inspired by the same objects and questions: What inspiration can (trainee) teachers draw from this structure?

These questions will again be explored in small groups to encourage a variety of possible answers. The background of each selected video will also be presented and discussed, in order to uncover the "hidden" relationships between their content and related issues in historical research. The soundtrack of the videos is in French, but English and Portuguese subtitles will be provided. Discussions will be held in English. Complementary materials related to the video clips will also be presented during the workshop.

Histoire des mathématiques à l'université Marie et Louis Pasteur (Franche-Comté, France) : un exemple de pratique

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3

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Nous présenterons l'enseignement d'histoire des mathématiques en L3 mathématiques (20/21 ans) destiné aux futurs professeurs du second degré (cours spécifique) ou aux futurs professeurs des écoles (éléments dans un cours de mathématiques).

Le contenu du cours "Histoire des mathématiques" (36 heures) est motivé par les contenus des programmes parus au *Bulletin Officiel* du 22 janvier 2019. Nous en présenterons les objectifs visés, le contenu, les modalités d'évaluation et les sources utilisées.

Le contenu du cours "Fondements mathématiques pour l'école primaire" (54 heures) s'adresse à des étudiants de mathématiques, de biologie, de sciences physiques. L'histoire des mathématiques est source de réflexion pédagogique. Par exemple, l'étude de numérations anciennes alimente l'étude des nombres entiers ainsi que celle des algorithmes des opérations enseignées à l'école élémentaire ; la présentation de différents points de vue sur la géométrie (Euclide, Descartes, Klein) fait réfléchir sur les contenus à enseigner.

Après une présentation du contenu et des sources utilisées, ce sera l'occasion d'échanger sur les pratiques, en particulier sur les ressources.

Références utilisées avec les étudiants et les étudiantes

Marc Moyon & Dominique Tournès (dir.) (2025). *Passerelles : enseigner les mathématiques au cycle 3*, Bouc-Bel-Air: ARPEME (1ère impression, 2018). `https://publimath.fr/iwh25001/`

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Les classiques Kangourou (Fibonacci, Al-Khwarizmi, les *Neuf Chapitres*, Euclide, Ozanam, Descartes, Alcuin).

Historical sources and classroom culture: Obstacles and opportunities concerning *Liber abbaci* in secondary school

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2

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This workshop is based on the results of a study on the ways of using some problems from Fibonacci's *Liber abbaci* to develop proportional reasoning in secondary school. The experimental part of this research involved students between grades VII and X. The initial working hypothesis was to develop the idea of proportionality in VII-grade students by using historical artifacts: specifically, the original text of problems of Liber abbaci (Boncompagni, 1857) translated into Italian (`www.progettofibonacci.it`); diagrams extracted from the manuscript of *Liber Abbaci* (Biblioteca Nazionale Centrale, Conv. Soppr. C.1. 2616, ff. 1r-214r); manipulative artifacts referring to the Euclidean geometric inspiration of Fibonacci proportionality. The selected problems were from *Liber abbaci*, Chapter XII, Section 3, where the *tree rule* (*regula arboris*), better known as *false position*, is explained by using the diagram today called *rule of three* (*regula universalis*). By using these artifacts, I attempted to provide students with a pre-algebraic method (without the use of equations) for solving first-degree problems. The results of various experiments (later also carried out for comparison in the first two years of high school) suggested some reflections about the obstacles and opportunities inherent in the use of *Liber abbaci* in secondary school. Progressively, the classroom culture (the current availability of equations and, later, functions, with the related semiotic tools: algebraic symbols, cartesian graphs) and the cultural distance (current dynamic ways of thinking about the relationships between variables – e.g. space and time) were recognized as the main obstacles for the hypothesized learning path. Documents (tasks including historical texts and some students' representative solutions) will be available in English for discussion.

These obstacles have led, through studies in the historical-mathematical and cognitive-didactic domains, to a deeper understanding of the socio-cultural context of *Liber abbaci*, and to the hypothesis of a use of some problems from it, which was different from the initial working hypothesis and different in different school levels. In lower secondary school, some experiments showed that these problems could be used as a tool for the initial approach to the historical dimension of mathematics and for conceptual strengthening. In high school, the results showed that *Liber abbaci* could be used both as a tool for a study of this historical dimension from an interdisciplinary perspective, and as an occasion for in-depth reflections about different ways of solving proportionality problems and related semiotic systems.

In order to understand the reasons behind the hypothesis that the identified obstacles may become opportunities for historical distancing and conceptual reinforcement, some problems from *Liber abbaci* (emptying a container, and pursuit) will be re-proposed using the same methods they were posed in the classrooms (work in pairs, the class divided into two groups), in order to compare different types of resolution (false position, Cartesian graph) and explore the related mathematical content. Then, word problems inspired by those from *Liber abbaci* will be solved by participants, and their learning effectiveness will be discussed.

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Inexhaustible heritage of Johannes Kepler: three inspirations for high school mathematics

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The workshop will be based on my own experience with implementing historical material in mathematics classroom. I teach mathematics at Johannes Kepler Gymnasium, secondary school in Prague named in an honour of this astronomer; it is therefore natural to try to introduce some of his concepts into curricula not only of physics, but also of mathematics. During last years, I have developed tasks and worksheets in three different topics: regular polyhedra (in lessons of stereometry), conics (in lessons of analytic geometry in plane) and plane tessellations (in lessons of elementary geometry, especially regular polygons). In all of them, we can use directly or indirectly the historical material of Kepler's writings. I bring materials used in classroom and we can discuss their use, reactions of students and potential for extension.

The workshop will be organised in following steps:

1. Introduction to work and life of Johannes Kepler (1571?1630) in connection with his stay in Prague and the overview of his mathematical investigations which can be implemented on high school level of mathematics.
2. Working group on the analysis of worksheets for students concerning investigation of regular polyhedra based on Kepler's *Mysterium cosmographicum* (1596). Discussion.
3. Working group on tasks for classroom based on Kepler Laws (*Astronomia nova*, 1609), regarding conic sections and power functions. Discussion.
4. Working group on regular and semiregular plane tessellations in connection with Kepler's *Harmonices mundi* (1619). Discussion.
5. Conclusion, final remarks and inspiration for implementing other parts of Kepler's work into mathematical education. E. g.: Kepler's conjecture, *Nova stereometria doliorum vinariorum* (1615), six-cornered snowflake.

Participants: secondary school teacher, university students, researchers.

Age of participating students: from 13 to 19 years, depending on their curricula and abilities

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Inverse Tangent Constructions in Dynamic Geometry: A Formula? Free Approach

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2

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The historical development of Calculus (which spanned from 17th to 18th century) was deeply intertwined with geometric and mechanical methods for solving differential equations, long before its later formalization through limits. Inverse-tangent problems offer a particularly rich example of this tradition. Yet a natural question arises: to what extent can such problems be meaningfully explored today without relying on symbolic calculus?

In this workshop, we present an approach to solving inverse-tangent problems within a dynamic geometry environment. Our proposal builds on the historical work of Bos (1988) and Tournès (2009), as well as on foundational contributions (Milici 2015, 2020). We aim to show how these geometric/mechanical ideas can be re-implemented in Dynamic Geometry Systems (DGS)–in particular GeoGebra–through easy and visually driven constructions.

We revisit several classical problems–such as the construction of the tractrix, the exponential curve, and devices that graphically integrate functions (integraphs)–using a minimal scripting strategy in GeoGebra. Our aim is explicitly to avoid analytic formulas: we do not compute the solution of a differential equation. Instead, we employ a single line of scripting (via the "SetValue" command) to update the evolving configuration in a way that preserves the spirit of the mechanical devices.

After a short introduction to the essential scripting components and the historical problems, participants will engage hands-on with inverse-tangent constructions carried out without symbolic calculus. This practical work will allow us to revisit epistemological questions that often remain implicit when problems are approached purely symbolically or computationally. For example: What conceptual principles truly underpin these constructions? Which assumptions shape the behaviour of an inverse-tangent mechanism when different tools are used–such as dragging a point versus employing a wheel-based direction device (Dawson et al., 2021)?

Participants are invited to bring their own device with GeoGebra installed, and it is recommended that they already have basic familiarity with the software (otherwise they could work in couples). In the final part of the workshop, we will collectively reflect on how such a formula?free, historically grounded approach can be implemented in classroom settings.

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Luisa Volterra D'Ancona's Biomathematical Research as a Resource for Mathematics Education

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The workshop is situated at the intersection of the History of Mathematics, Biomathematics and Mathematics Education, drawing inspiration from the still little-known figure of Luisa Volterra (1902?1983). Daughter of Vito Volterra and wife of the biologist Umberto D'Ancona, Luisa Volterra was an intelligent and determined woman who contributed to early studies that paved the way for the mathematical modeling of ecological systems, closely associated with the well-known predator-prey model.

The workshop adopts storytelling as a core pedagogical methodology to foster engagement, identification, and sense-making, using the life and scientific work of Luisa Volterra as a narrative and epistemic thread. Storytelling is not treated as a purely introductory device, but as a structured pedagogical approach that supports conceptual understanding and historical contextualization, and is systematically integrated with technological tools and digital learning environments.

Participants work with interactive digital timelines to situate Luisa Volterra's research within its historical and scientific context and engage in the reading and discussion of primary historical sources, in particular, selected letters exchanged between Luisa Volterra and her collaborators. These materials are used to reconstruct the intellectual, cultural, and institutional framework in which early biomathematical "developed, supporting a historically grounded understanding of " problems, methodological approaches, and modeling choices.

This historical dimension is closely connected to hands-on laboratory activities carried out in advanced computational environments, such as Maple, which are used to explore and model population dynamics. Through this integration, participants move from historical narration to formalization and simulation, bridging qualitative reasoning and quantitative modeling. This fosters reflection within the field of environmental education, showing how historical case studies, once translated into mathematical models, can support insights into ecosystem protection, sustainability, and the impact of human and natural factors on ecological systems, within the framework of civic education.

The workshop explicitly proposes an integration between past and present, combining the History of Mathematics and Science with contemporary digital technologies to support meaningful learning processes in Mathematics Education. It is designed for both in-service and pre-service teachers, with a focus on transferable instructional design strategies. The teaching activity is primarily targeted at students in the final years of upper secondary school, while remaining adaptable to different educational levels, curricula, and classroom contexts.

Problematizing the Pythagorean Theorem through the Practices of the Pythagorean Community

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The Pythagorean Theorem is a mathematical property widely explored in Brazilian basic education, and Pythagoras himself is a figure surrounded by legends and mysteries. It is commonly assumed that he discovered this property and was the first to prove it. However, scholars of Ancient Greece have proposed new interpretations of the mathematics of that period (e.g., Burkert, 1972; Unguru, 1975), even questioning whether the Pythagorean Community actually worked with geometry (Netz, 2022). It is believed that Pythagorean mathematical practices were more closely related to figurate arithmetic (Roque, 2012). Given their wide reach within Brazilian schools, textbooks reveal which images are constructed about Pythagoras, the Pythagorean Community, and the Pythagorean Theorem (Bernardes, Moustapha-Corrêa, Amadeo, 2025). Despite authors' efforts to include a historical perspective in mathematics (Amadeo et al., 2025), many myths already refuted by historians continue to be perpetuated.

In this workshop, we will explore a practice of the Pythagorean Community through what we call figurate numbers. We will examine what figurate numbers were, some of their properties, and how the so-called "Pythagorean Theorem" was understood by the Pythagoreans–as a numerical rather than geometric relation. We expect participants to recognize that what we call the Pythagorean Theorem may have distinct statements and purposes depending on its historical and cultural context. This workshop is part of a broader activity that also addresses Euclid's practices in Books I and II of the *Elements*, where the theorem is used to geometrically add squares.

Our proposal targets pre-service and in-service mathematics teachers and others interested in the integration of history and mathematics teaching. It is organized as follows: (1) an activity inspired by the practice of the Pythagorean Community, involving triangular and square numbers and the determination of Pythagorean triples (activity guide available at [https://drive.google.com/file/d/12iVm3-rWtOIGArpaySS6QeD_UvCh1J56/view?usp=sharing]), using chickpeas provided by us, followed by a discussion on myths surrounding Pythagoras and the theorem; (2) a debate on how the Theorem is typically taught in the participants' countries, followed by a presentation on the Brazilian approach; and (3) a discussion on ways to integrate history and mathematics teaching to promote reflection on mathematical content and school practices.

Our aims are to contrast a historical practice with the usual algebraic approach in Brazilian textbooks; to deconstruct myths related to this historical episode; and to foster a critical view of how history appears in educational materials. We emphasize the importance of studying the development of mathematical knowledge in teacher education. By the end, participants will have a concrete example of how integrating history and teaching can enrich both teachers' and students' understanding of mathematics.

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Proof with History

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3

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The history of mathematics is rarely present in mathematics classrooms, as well as in the many resources available for the teaching and learning of this subject. It is widely acknowledged that both initial teacher education and in service professional development devote little attention to this topic. However, the current *Aprendizagens Essenciais de Matemática* for upper secondary education (Direção-Geral da Educação 2023) explicitly highlight the importance of the History of Mathematics in the evolution of society. In this sense, the History of Mathematics is not conceived as a curricular topic, but rather as a means of establishing intramathematical connections between different topics, concepts, or representations within mathematics. It can thus support students? understanding, foster awareness of the historical development of concepts over time and illustrate how these concepts contribute to problem solving. In this workshop, we present and discuss several relevant topics in the History of Mathematics that enable connections between geometry, arithmetic, and algebra in the proof of theorems (Dreyfus et al. 2012).

Since 1740, Leonhard Euler (1707?1783) established a new branch of number theory (Debnath 2016), known as additive number theory. This field focuses on the study of partitions of natural numbers in various forms and can, as will be shown, also be related to polygons. Our aim is to show that many elementary properties of regular n-gons can be proved using partitions of the number n. To that end, we consider triangles inscribed in the circumcircle of the n-gon. This is achieved by partitioning the number n into three natural numbers n_1, n_2, and n_3 , such that $n = n_1 + n_2 + n_3$. This partition translates into a geometric action by dividing the n equal arcs of the circle into three successive parts of n_1, n_2, and n_3 arcs. By joining the three corresponding division points, we obtain a triangle formed by sides or diagonals of the n-gon. Examples include inscribed isosceles triangles. As far as we know, this approach has not yet been applied in school curricula and shows that complicated trigonometric reasoning sometimes can be reduced to a simple arithmetical one.

The proof that the two diagonals of a regular pentagon intersect in the golden ratio is frequently used to illustrate the application of triangle similarity. Recognising similar triangles is not particularly difficult, but what is often lacking is a systematic method. We show that our new approach to polygons, has the advantage of providing a systematic and generalisable proof, grounded exclusively in an arithmetic action – the partition of the number of vertices n into three parts. This approach can therefore enrich the discussion of regular polygons in the mathematics classroom. Moreover, it offers an additional perspective on the additive number theory established by Euler. We particularly highlight the fact that additive number theory has gained significant importance due to its applications in combinatorics (a recurring topic in Mathematics Olympiads) and in other areas of discrete mathematics, (Debnath 2016).

Note: All materials will be made available to participants.

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Some reflections on the role of the table as an artifact in the mathematics classroom based on historical examples

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1

Problem-solving strategies are identifiable methods of approaching a task that are completely independent of the specific topic or subject. In 1945, mathematician and educator George Polya published his classic *How to Solve It*, which, perhaps for the first time, addressed problem-solving as a topic in itself. His strategies and recommendations have appeared and continue to appear in many resource books and school mathematics textbooks to this day.

One of the suggested strategies for problem-solving is to organize a list to collect data. Therefore, it is important for students to develop the ability to generate tables that assist them in the process of organizing data and facilitate the identification of patterns. To create meaningful tables, students must identify what data to focus on and how to organize it. Graphic organizers, in general, and tables, in particular, can help visualize concepts, organize and/or sequence information, systematize the problem-solving process, generate ideas, discover connections between ideas, compare and contrast ideas, and also assist in formulating conjectures.

The book *The History of Mathematical Tables: From Sumer to Spreadsheets* (Campbell-Kelly Croarken, Flood, Robson, eds., 2003) summarizes the technical, institutional and intellectual history of these mathematical artifacts generated from antiquity to those created with software at the end of the 20th century. It includes analysis and discussion of data tables such as those obtained from population censuses, professional tables such as actuarial tables, tables with astronomical observations and one of the first trigonometric tables such as Plimpton 322, from Babylon from the 18th century B.C.E. It can be deduced then, that tables are an essential part of the mathematical work of both those who create them and those who use them, and not only in modern times with the advent of tools such as the spreadsheet.

In this workshop designed for middle school or high-school mathematics teachers, they will actively explore some of the tables mentioned in that book and they will reflect upon the different roles these tables play in the development of mathematical ideas, in the discovery of patterns, and they will identify school opportunities to expose their students to these valuable experiences.

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The Multipurpose Tangent Solver: A Hands-On Journey through the History of Calculus

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We aim to introduce participants to a hands-on educational pathway for addressing key concepts in Calculus using a modular, history-based material artefact called the Multipurpose Tangent Solver (MTS, cf. Maschietto & Milici, 2025; Milici et al., 2025). The workshop is based on didactical materials tested with students in the final two years of secondary school (aged 18-19) and aims to support the integration of historical and epistemological insights into classroom practice through the methodology of the mathematics laboratory (Maschietto & Bartolini Bussi, 2011).

The MTS recollects a rich historical lineage of mathematical instrumentation (Bos, 1988; Tournès, 2009), tracing back to the 17th century with Leibniz's approach to Calculus. His vision resonates with earlier contributions by Christiaan Huygens, whose work on curves and mechanical devices embodied a deep interplay between geometry and instrumentation. These historical roots culminated in 18th-century geometric machines by Giovanni Poleni, later echœd in 19th-century integraphs designed to trace solutions to differential equations. In particular, the topics covered by MTS include tangents (direct/inverse problems), derivatives/antiderivatives, and transcendental functions (exponential curves).

From an educational perspective, the MTS is analysed through the lens of the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008), wherein an artefact may be deliberately employed by an expert to mediate mathematical meanings through a structured didactic intervention. This theoretical framework supports the design of tasks that guide learners from the manipulation of the MTS toward the understanding of the mathematical meanings embedded in it.

In the workshop, participants will be organized into small groups and provided with MTS. They will explore the artefact through guided tasks organized in worksheets and participate in collective discussions. The aim is to reflect on the mathematical meanings mediated by the artefact and on the semiotic activity produced during the working session (gestures, drawings, and verbal expressions).

In the final part of the workshop, we will share and reflect with the audience on the results obtained from classroom experiments with students and open a general discussion on the educational potential of the MTS and its integration into teaching practices and dissemination activities.

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Un exemple de mise en œuvre de la méthodologie du groupe AHMES : former à l'enseignement du périmètre et de l'aire du disque

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3

Le groupe AHMES (Apports de l'histoire des mathématiques aux enseignants du secondaire) de l'IREM de Clermont-Ferrand (France) a pour projet d'introduire une dimension historique dans le parcours de formation professionnelle des enseignants (formations initiales et continues) ainsi que dans l'enseignement des mathématiques dans le secondaire (collège et lycée). L'idée est de montrer comment l'enseignant peut enrichir sa compréhension des concepts contenus dans les programmes et se familiariser avec des aspects didactiques de sa discipline à travers un éclairage historique. L'objectif final étant de lui permettre de repenser et de perfectionner son enseignement en anticipant davantage les difficultés des élèves, en comprenant les obstacles éventuels et en étant mieux à même de motiver les notions nouvelles auprès des classes.

Depuis sa création en 2015, le groupe AHMES conçoit des formations mises en œuvre dans le cadre du Master des Métiers de l'Enseignement et de la Formation et intervient régulièrement dans le plan académique de formation auprès des enseignants. Pour cela, il a mis au point une méthodologie de formation basée sur une problématique d'enseignement précise et conduisant à un double questionnement didactique et historique. Cette méthodologie est en cours de conceptualisation.

L'atelier proposé ici abordera une problématique d'enseignement située au niveau du collège (cycles 3 et 4) autour de la "mesure du cercle" – pour reprendre le titre de l'ouvrage d'Archimède – autrement dit, concernant la circonférence du cercle et l'aire du disque. Il présentera une action de formation continue à l'intention des professeurs de collège ; action qui sera expérimentée par les participants afin de leur faire voir la démarche méthodologique du groupe. Les supports de formation seront mis à disposition (extraits de manuels scolaires français, textes historiques, matériel pédagogique pour les classes...) pour une appropriation individuelle et des temps d'échange et de discussion collectifs seront ménagés une fois ces supports éprouvés.

L'atelier sera essentiellement conduit en français (un diaporama en anglais sera également utilisé) mais les échanges et la discussion pourront avoir lieu en anglais.

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D'autres sources primaires et secondaires ont été utilisées pour la préparation et la conception de l'action de formation qui sera décrite dans cet atelier, mais les éléments bibliographiques précédents sont ceux qui ont servi de support aux activités soumises aux formés.

Il est possible de consulter le travail du groupe AHMES sur la page de site de l'IREM de Clermont-Ferrand : `https://irem.uca.fr/groupes/ahmes`

Understanding ratio through cultural connections: braiding Euclid's *Elements*, Book V *hóroi* to a narrative account of primordial measure

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The concept of ratio occupies a pivotal position in mathematics education, yet it is frequently reduced to a numerical operation and introduced only after arithmetic procedures are firmly established. Yet Édouard Séguin's rods for the cognitively disabled and Maria Montessori's educational "sensorial" materials for early childhood involve geometrical ratio leaning on intuition.

Recently, a piece of children's literature written by Pamela Vale, Mama Khanyi and the Pots (2019) (`https://www.associazionetokalon.com/i-vasi-di-mamma-khanyi_en/`), inspired by a story by José Luis Cortina, has bound together measure, ratio, and rational numbers in a primordial fictional scenario illustrated by Carmen Ford.

Following Enrico Giusti's connection between the primitive concepts of point and straight line, and natural number in action before writing, and the *Elements*' Book I and Book VII *hóroi* (Giusti 1999), we have created a workshop inspired by Book V *hóroi* I–VI. This work is also informed by recent prehistoric, archaeological, and ethnographical research on measurement (Morley & Renfrew 2010, Crease 2011, Lugli 2019), and combines the reading of Vale's story – also thanks to her own questions and problems posed to the reader – with hands-on activities.

Moreover, scholarship on the modern "arithmetization" of ratio was also inspiring (Malet 2017).

The workshop was designed for children in primary school and adapted for middle school and for prospective primary and middle school teachers, who also read Euclid's hóroi. Ratio thus appears not as a numerical calculation (division, in fact), but as a structural relationship embedded in material practices of measuring. These practices involve geometrical comparison of lengths and other magnitudes, and are crucially linked to the human body and to social processes such as sharing, agreeing, and harmonizing.

Our working hypothesis is that this approach makes it possible to address the issue of meaning (as opposed to rigor, following René Thom) with respect to ratio, a key concept in mathematics and in modern science (measurement, proportionality, trigonometry, rational numbers). This holds true even in its present, mainly numerical guise: geometrical ratio is still active in graphic representations and, more broadly, in understanding beyond numbers and formulae.

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Exhibitions

Experimenting the tangibility of Tangent with Historical Mechanisms

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In this exhibition, we present various reconstructed historical machines related to the material solution of the inverse tangent problem. We follow the historical evolution of ideas and instruments from the origin of the problem (1670s) through the 18th century, featuring machines by Perrault, Jacques Bernoulli, Perks, and Poleni. Besides them, we propose some devices to draw conics with string or load in order to compare them, exploring similarities and differences. Artefacts come along with an excerpt of primary and secondary sources, a video presenting the reconstruction of the Poleni?s machines, and panels guiding visitors through such a tangible approach to the tangent. Our perspective is that historical artefacts can serve as powerful tools for didactical reflection and conceptual understanding (from abstract formalism to embodied mathematical experience), as we experimented with teachers and university students with a similar selection of material and textual sources.

See the workshop "Experimenting the tangibility of Tangent with Historical Mechanisms".

Émile Borel, a Mathematician in the Plural

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In 2021, at the occasion of the 150th anniversary of the birth of Émile Borel (1871?1956), the Institut Henri Poincaré in Paris organized an exhibition dedicated to its founder. It was designed by six historians of science (A. Bernard, M-C. Bustamante, M. Cléry, C. Ehrhardt, H. Gispert, L. Mazliak). Entitled "Émile Borel, a mathematician in the plural," the exhibition presents Borel's mathematical work in diverse aspect: disciplinary, but also at the interface of applications, particularly in the humanities, as well as didactic and institutional. Moreover, the panels could be used as a support for visits by secondary school students accompanied by their science teachers. This last aspect represented a challenge, insofar as it was not easy to convey the modernity of Borel's work to an audience rather distant from the issues concerned. A demanding task of popularization had to be carried out, both scientific and historical, aimed at placing Borel and his work in the social and political context of his time.

During the six months the exhibition was on display at the library, the Institute's communication department organized class visits; some were guided by us. Short practical internships for secondary school teachers were also organized, requiring the expertise of the historians, science mediators, mathematicians and trainers who organized the exhibition. Based on a series of informative texts about Borel and on the understanding of the choices of the exhibition's scientific committee, the trainees were led to develop a coherent pedagogical project in which the visit of the exhibition would be included.

All this was only the beginning of the exhibition's adventure. Following its presentation at the IHP, requests came for it to be loaned out and presented in other places, first in France and then in several other countries through translations. The exhibition currently exists in seven languages (including Portuguese, as can be seen during the present conference). The loan system offered by the IHP's communications department is simple and particularly effective. The exhibition was moreover an occasion for some talks on Borel or other figures related.

See the workshop "The Borel exhibition at the IHP: historical and educational issues".

Useful Information

How to get to Aveiro?

`https://esu10.sciencesconf.org/resource/page/id/14`

- By Plane (to Lisbon or Porto) The international airports are Airport of Porto (at about 70 km, north of Aveiro) and Airport of Lisbon (at about 250 km, south of Aveiro). Both airports have regular scheduled flights to and from all main European destinations as well as major world cities.
- By Train (Since the COVID pandemic, there are currently no direct trains from Portugal to Madrid or Paris)

  - From Lisbon to Aveiro:
    There are comfortable trains arriving to Aveiro hourly from the Lisboa-Oriente station (you can arrive there from the airport by metro (use the direct Red line or taxi). The train journey takes about 2 hours and 20 minutes.
    Lisbon Airport – metro/taxi $\rightarrow$ Lisboa Oriente – train $\rightarrow$ Aveiro

- From Porto to Aveiro:
  There are comfortable trains arriving to Aveiro hourly from Porto-Campanhã station (you can get there from the airport by bus, taxi or metro (use the direct Violet metro line from the Aeroporto). The train journey takes about 1 hour.
  Porto Airport – metro/bus/taxi → Porto-Campanhã –train → Aveiro

- By Bus There are express buses arriving every hour both from Lisbon and Porto. See Rede Expressos or FlixBus.
- By Car The main motorways passing through Aveiro are: highway A1 (main portuguese highway between Porto and Lisbon), A29, A25. After arrival, search for directions indicating "Universidade". These will drive you into the main university campus.

In Aveiro, at the university.

- **Auditório Carlos Borrego** (in Dept. of Environment and Planning): Registration (1st day), Opening and Closing sessions, Plenary Lectures, Panel Discussion and *Mathematical Circus* (with round table) on Monday.
- **Departamento de Matemática** (Dept. of Mathematics): Oral Presentations, Short Oral Presentations (with Posters), Workshops and Breaks. The rooms 11.1 are on the ground floor, room 11.2.25 is on the first floor.
- All lunches are taken at the **Restaurant**.

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